DOCUMENT RESUME

ED 406 204 SE 059 919

AUTHOR Kestner, Michael; And Others
TITLE Geometry Strategies, Grades 6-8.
INSTITUTION North Carolina State Dept. of Public Instruction, Raleigh.
PUB DATE [97]
NOTE 204p.
AVAILABLE FROM North Carolina State Department of Public Instruction, 301 N. Wilmington Street, Raleigh, NC 27601-2825.
PUB TYPE Guides - Classroom Use - Teaching Guides (For Teacher) (052)

EDRS PRICE MF01/PC09 Plus Postage.
DESCRIPTORS *Geometry; Intermediate Grades; Junior High Schools; Mathematics; Measurement; Middle Schools; Models; Problem Solving; Thinking Skills
IDENTIFIERS *Mathematics Activities

ABSTRACT

This document contains informal geometry explorations that provide students with opportunities to investigate the environment from a geometric perspective, construct connections between related mathematics and other content areas, and solve problems in a geometric context. The activities provide experiences that develop curiosity, understanding, and problem solving skills. Activities, exploration, and experimentation include: (1) translating ideas into mathematical language and symbols; (2) learning to make reasonable estimates; (3) developing independence in solving meaningful problems; (4) developing measurement skills; (5) constructing and interpreting tables, charts, and graphs; and (6) examining notions of elementary probability and statistics. (JRH)

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Geometry Strategies

Grades 6-8

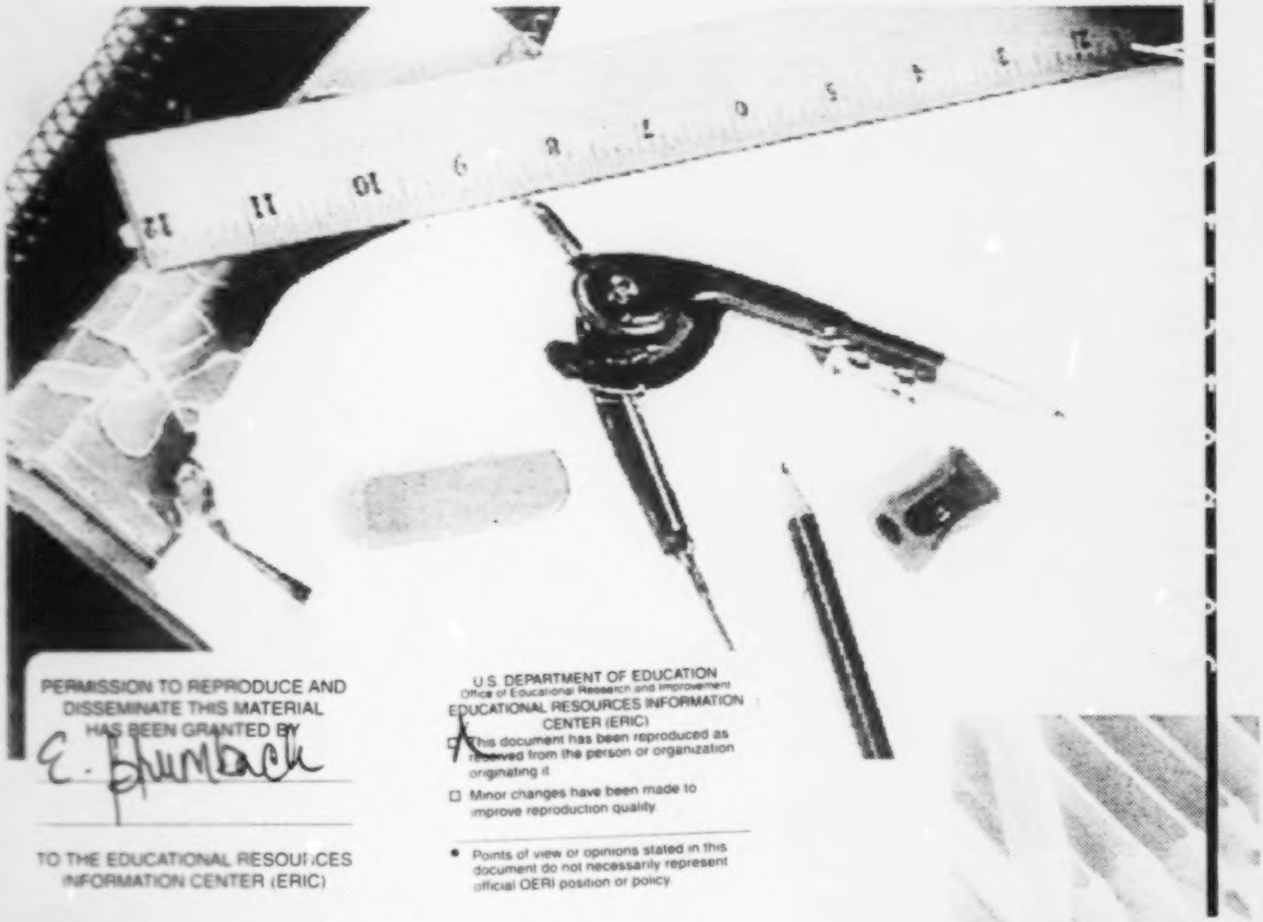

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Table of Contents

Introduction 1

Sixth Grade Strategies

Activities by Objectives 1
Blackline Masters 13

Seventh Grade Strategies

Activities by Objectives 1
Blackline Masters 15

Eighth Grade Strategies

Activities by Objectives 1
Blackline Masters 20

Appendices

A: Geometric Thinking Development 1
B: Glossary of Geometric Terms 8

Acknowledgement

Some activities are drawn from *Mathematics Achievement through Problem Solving (MAPS PLUS)*, a year long mathematics program developed at Florida State University, Tallahassee, Florida.

For more information contact:

Grayson H. Wheatley
219 Carothers Hall
Florida State University
Tallahassee, FL 32306-3032
(904) 644-8497

Development Teams and Contributors

The Department of Public Instruction extends its gratitude to the many North Carolina teachers, and other members of the educational community who willingly shared their ideas and expertise in developing this material. Teachers in the 1994 Middle School Mathematics Summer Institute joined Iris Marbet and other members of the education community in writing the geometry activities book for teachers to use with students in grades 6-8.

Grade Six Team
Kathryn Allfred
Barbara Coddington
Rebecca Collins
Dianne Hobbs
Terri Hunter
Barbara James
Patricia Johnson
Peggy Mcllwain
W. C. Musten, Jr.
Holly Smith

Grade Seven Team
Deon Andrews
Shelia Brookshire
Jeannie Galluzzo
Russie Hathaway
Kay Laster
Bonnie Mashburn
Cathy Mason
Brenda Mckinnon
Susan Murray
Suzanne Parris
Louise Uziel

Grade Eight Team
Stella Farrow
Janet Jenkins
Ann Mabry
Jan Maness
Helen Owen
Barbara Yost

Community Educators
Patricia Baker
George Bright
Gilbert Casterlow
Sandra Hodges
Bonnie Johnston
Linda Patch
Sherron Pfeiffer

The final document was prepared and edited by Michael Kestner, Linda Love, Toni Meyer, Linda Patch and Bill Scott.

The Mathematics/Science section is interested in your reaction and suggestions to the material in this book. Please take a moment to respond to the following questionnaire and return it to the Department of Public Instruction by mail or FAX.

1. What is the most helpful part of these materials? What do you like best?

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2. What suggestions do you have if similar materials are prepared for grades six-eight?

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3. Circle the grade level you teach: 6 7 8

4. Circle the number of years teaching: 1-8 9-15 16-25 More than 25

5. If you would like to become a part of future writing teams, please give us your:
   Name ______________________________
   School ______________________________
   Address ______________________________
   Phone # ______________________________
   Home Phone # ______________________________

6. Other comments?

____

____

____

Optional:
Name ______________________________
School ______________________________
School System ______________________________

Linda Patch, Consultant
Department of Public Instruction
301 N. Wilmington Street
Raleigh, N. C. 27601-2825
Phone: (919) 715-2225
Fax: (919) 715-0517
E-mail: lpatch@dpi.state.nc.us

Geometry in the Middle Grades

> *Geometry is grasping space . . . that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it.* (Freudenthal 1973, p. 403)

Geometry is all around us. Geometry can be found in the structure of the solar system, in rocks and crystals, and in plants and animals. It is found in the design of our buildings, the investigation of nature, appreciation of art. Understanding geometry increases a person's level of appreciation of the world in which we live.

The study of geometry helps students represent and make sense of the world. Geometric models provide a perspective from which students can analyze and solve problems, and geometric interpretations can help make an abstract (symbolic) representation more easily understood. Many ideas about number and measurement arise from attempts to quantify real-world objects that can be viewed geometrically. For example, the use of area models provides an interpretation for much of the arithmetic of decimals, fractions, ratios, proportions, and percents.

Most geometry that is taught in grades 6-8 can be referred to as informal geometry. Informal geometry is experiential. It provides students with opportunities to investigate the environment from a geometric perspective, to construct connections between related mathematics and other content areas, and to solve problems in a geometric context. Sometimes it is artistic. Sometimes it involves learning the name of a shape or a property that has been observed. Sometimes it includes building a prototype or taking one apart. Sometimes it is solving a shape puzzle. Sometimes it is seeing how things are similar or different or if an observation can be generalized.

Learning geometry should be fun and challenging for middle school students. To achieve this they must be engaged in meaningful activities. They must be provided experiences that develop curiosity, understanding, and powerful problem solving skills. Use of physical models, connecting measurement to geometric concepts, applying mathematical procedures to realistic situations, and allowing mathematical creativity are key components of effective instructional strategies.

Students discover relationships and develop spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures. Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments. The informal exploration of geometry can be exciting and mathematically productive for middle school students. At this level, geometry should focus on investigating and using geometric ideas and relationships rather than on memorizing definitions and formulas.

Sample Explorations

Activities which illustrate an intuitive nature of learning:

- On a geoboard, have students construct right triangles, squares, rectangles, and other polygons.

- Pair students and have each construct a polygon on a geoboard without revealing the shape to the partner. Each describes properties of the polygon until the partner is able to identify the figure.

- Help students draw and label various three-dimensional shapes. From their drawings select a group of shapes to display to the class. Now either show a model of a solid shape or select a drawing from among the students. Students should identify from within the group of drawings a drawing that has similar properties of the selected drawing or model. Have them defend their selection by stating the property(ies) that make them similar. Repeat the process.

- On the overhead projector show students a list of clues for a shape, uncovering the clues one at a time. Tell students to inform you when enough clues are given for them to know for sure what shape it is. Otherwise they should ask for another clue.

  Example:
  It is a closed figure with four straight sides.
  It has two long sides and two short sides.
  It has a right angle.
  The two long sides are parallel.
  It has two right angles.
  The two long sides are not the same length.
  The two short sides are not the same length.
  The two short sides are not parallel.
  The two long sides make right angles with one of the short sides.
  It has only two right angles.

- Ask students to suppose they have a quadrilateral with opposite sides congruent. Must the opposite sides be parallel? What is the converse?

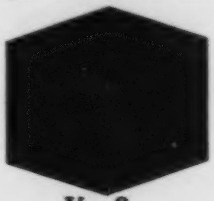
Area and volume

What happens to the area of a square when the length of the side is doubled?

What happens to the surface area and the volume of a cube when the length of a side is doubled?

Investigations of increase in two- and three-dimensional models foster an understanding of the relationship between growth rates of linear measures, areas, and volumes of similar figures. These ideas are fundamental to measurement and scientific applications.

$V = 1$
$SA = 6$



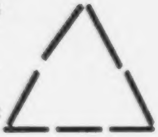
$V = 8$
$SA = 24$

$L = 1$
$A = 1$

$L = 2$
$A = 4$

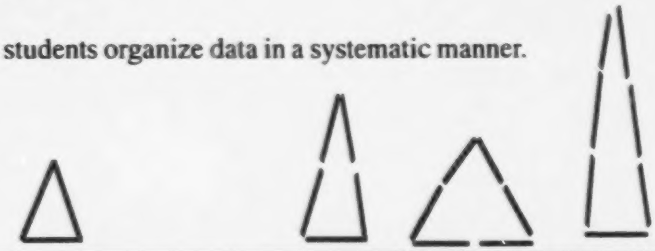
Triangles

You are given a pile of toothpicks all the same size. First, take three toothpicks. Can you form a triangle using all three toothpicks placed end to end in the same plane? Can a different triangle be formed? What kinds of triangles are possible?



Now take four toothpicks and repeat the questions. Then repeat with five toothpicks, six toothpicks, and so on.

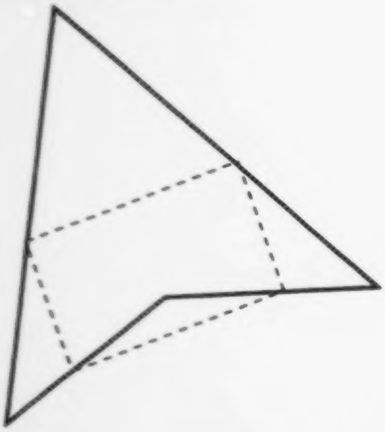
The following table helps students organize data in a systematic manner.

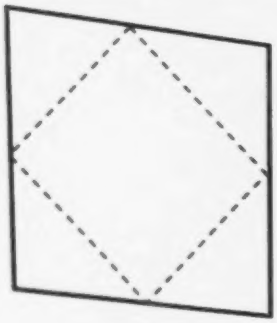


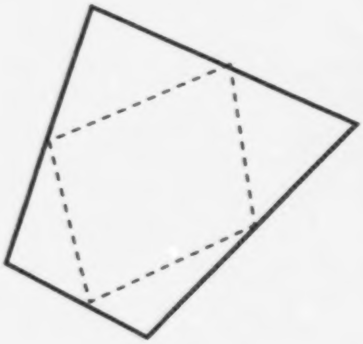
# toothpicks	3	4	5	6	7	...
is triangle possible?	Y	N	Y	Y	Y	...
# of triangles	1	0	1	1	2	...
Kind of triangles	equilateral		isosceles	equilateral	isosceles	...

In this activity, students find that the sum of the measures of two sides of a triangle must be greater than the measure of the third side. This activity also reinforces the classification of triangles by sides and angles.

Quadrilaterals







Students can explore what happens when they connect the midpoints of the sides of several quadrilaterals. Their investigations can prompt them to ask such questions as: What kind of figure results? How does the area of the new figure compare to that of the quadrilateral? What quadrilateral would you start with so that the new figure is a rectangle? A square?

Curriculum Goals

The primary goal of mathematics education is to ensure that every student will become mathematically literate. Literacy implies that a student (1) is a problem solver; (2) is confident in his/her ability to do mathematics; (3) is able to communicate mathematically; (4) is able to reason mathematically and (5) values mathematics. Mathematical literacy is necessary for the layperson as well as for the professional who chooses a career in mathematics or in a field which makes use of mathematics.

The mathematics program in North Carolina is broader and more inclusive than in the past. It is developed with more vocabulary, facts and principles; more than the ability to analyze a problem situation; more than understanding of the logical structure of mathematics. Mathematics must provide students with knowledge which will enable them to distinguish fact from opinion, relevant from irrelevant data, and experimental results from proven theorems. Curricula should stimulate curiosity so that students will enjoy exploring new ideas and create mathematics which is new for them, even though it has been developed by others. The mathematics program must produce students who know how to learn mathematics, enjoy learning mathematics, and are motivated to continue their learning. All of the mathematics courses have the goal of developing both competence and confidence in mathematics for all students.

In grades 4-6 students move to a “skill establishment” stage. Activities, exploration and experimentation include:

- translating ideas into mathematical language and symbols.
- learning to make reasonable estimates.
- developing independence in solving meaningful problems.
- developing measurement skills.
- constructing and interpreting tables, charts and graphs.
- examining notions of elementary probability and statistics.

In seventh and eighth grades, the skills development in the elementary grades are reviewed and extended. The program offers students of all ability levels the opportunity to develop a better understanding of numbers, improve their ability to reason, and be exposed to some exciting new areas of mathematics. The topics emphasized at these grade levels are: operations on rational numbers, beginning algebra, informal geometry, measurement, graphs, scale drawings, elementary probability and statistics, and problem solving.

The use of calculators and computers to enhance and enrich the mathematics program is encouraged. Computers and calculators should be integrated into the mathematics curriculum in imaginative ways. In addition to their use with numerical calculations, they should be used to graph equations, display data, clarify situations as they enable a student to attack a problem from a different point of view. Several computer programs will allow students to investigate shapes and sizes from an experimental approach. Technology is also making that ability available on calculators.

North Carolina Standard Course of Study, 1995

Geometry and Patterns Standards, 5-8 Identified in the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics*

STANDARD 8: PATTERNS AND FUNCTIONS

In grades 5-8, the mathematics curriculum should include explorations of patterns and functions so that students can:

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships using tables, graphs, and rules;
- analyze functional relationships to explain how change in one quantity affects change in another;
- use patterns and functions to represent and solve problems.

STANDARD 12: GEOMETRY

In grades 5-8, the mathematics curriculum should include the study of the geometry of one, two, and three dimensions in a variety of situations so that students can:

- identify, describe, compare, and classify geometric figures;
- visualize and represent geometric figures with special attention to developing spatial sense;
- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- understand and apply geometric properties and relationships;
- develop an appreciation of geometry as a means of describing the physical world.

STANDARD 13: MEASUREMENT

In grades 5-8, the mathematics curriculum should include extensive concrete experiences using measurement so that students can:

- extend their understanding of the process of measurement;
- estimate, make, and use measurements to describe and compare phenomena;
- select appropriate units and tools to measure to the level of accuracy required in a particular situation;
- understand the structure and use of systems of measurement;
- extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight/mass;
- develop the concepts of rates and other derived and indirect measurements;
- develop formulas and procedures for determining measures to solve problems.

Instructional Strategies

1. An eye opener activity at the beginning of every class helps students focus prior to the presentation of mathematics concepts.
2. Fact and vocabulary activities provide students an opportunity to be successful (during reviews), to strengthen their knowledge of mathematics (facts/vocabulary), and to respond openly in a group setting.
3. Take some time at the end of a lesson for reflections and making connections to future lessons.
4. Providing instructions/expectations prior to each activity improves the student's opportunities for success.
5. Rewarding students exemplary performances or active participation reinforces the importance of the learning process.
6. Target students who tend to be passive learners with activities designed to involve students as participants of a group (i.e. chanting, spelling vocabulary words).
7. Emphasizing note taking helps students focus on the activities.
8. Mental games (i.e. quick answers to problems involving a series of operations) prove challenging and offer a change of pace to the regular classroom routine.
9. The rapid transition from one activity to another keeps students constantly engaged.
10. Daily handouts can serve to reinforce facts, vocabulary, and concepts introduced during the class sessions.
11. Constantly reminding students of the importance of listening, following directions, being observant, and taking an active role in the class session's different activities reinforces expectations set forth at the beginning.
12. A diagnostic test administered at the beginning of the program can determine group needs and guide instruction.

Strategies for Taking Tests

1. If the score is based on the number of questions you answer correctly, you should answer every question (even if you have to guess). Do this before going to the next problem.

2. You should put a light mark next to questions that you may be able to work but that will require more time. If time permits, you can then try to work these. Erase the light marks before turning in your answer sheet.

3. Scan all of the answer choices. Some of the choices are designed to be those that are obtained when a common error is made.

4. Answer choices may give you an idea as to how to work a problem.

5. If the answers are spread apart from each other, estimating the result of a calculation can reduce the amount of time needed to obtain a solution.

6. Estimating can help to eliminate some answer choices that you determine cannot be correct.

7. Make sure that the question you answer is the question that was asked.

7. When first approaching a complex problem, put down your pencil, read the problem over to get the general idea, identify the desired unknown, and scan over the answer choices.

9. Some problems contain too much information. Weed out that which is unnecessary as you set up equations, and don't worry if you do not use all the information given.

10. Remember that an answer can be represented in more than one way. If you're fairly sure you did a problem correctly, check to see if the answer you got is the same as one of the answer choices given for the problem but given in a different form.

11. When you can eliminate one answer choice or more, try to make an educated guess and record an answer.

12. Make full use of the time allotted. If you have attempted all of the problems and have some time left, take the time to check your work.

13. If you are allowed to use scratch paper, use it. Do not recopy a problem from a test to the scratch paper if it is unnecessary.

14. Avoid spending too much time on any one question.

15. Make sure that the answer sheet is filled correctly. Be careful when erasing; take the time necessary to ensure that answers to problems worked are placed correctly on the answer sheet.

Questioning Techniques

Effective questioning is an art to be cultivated by all educators. These questions might be used informally during instruction for formal assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. Some suggestions about assessment questioning include:

General

- Prepare a list of possible questions ahead of time, but unless the assessment is very formal, be flexible. You may learn more by asking additional or different questions.
- Use plenty of wait time; allow students to give thoughtful answers.
- Make a written record of your observations.
- Build a collection of your own good questions.

Problem Comprehension Can students understand, define, restate, or explain the problem or task?

- What is this problem about? What can you tell me about this problem?
- How would you interpret that?
- Would you please explain that in your own words?
- What do you know about this part?
- Is there something that is not needed?
- What assumptions do you have to make?

Approaches and Strategies Do students have an organized approach to the problem or task? How do they record? Do they use tools (manipulatives, diagrams, graphs, calculators, computers, etc.) appropriately?

- Where could you find the needed information?
- What have you tried?
- What steps did you take?
- What did not work?
- How did you organize the information?
- Did you make a record?
- Did you have a system? a strategy? a design?
- Would it help to make a diagram or a sketch? a list? a table?

Relationships Do students see relationships and recognize the central idea? Do they relate the problem to similar problems previously done?

- Can you find a relationship?
- What is the same? What is different?
- Is there a pattern?
- Can you examine one part at a time? What would the parts be?
- Can you write another problem related to this one?

Flexibility Can students vary the approach if one is not working? Do they persist?

- Have you tried making a guess?
- Would another recording method work as well or better?
- What else have you tried?
- Give me another related problem. Is there an easier problem?
- Is there another way to (draw, explain, say, . . .) that?

Communication Can students describe or depict the strategies they are using?

- Would you please reword that in simpler terms?
- Could you explain what you think you know right now?
- How would you explain this process to a younger child?
- Could you write an explanation for next year's students (or some other audience) of how to do this?
- Which words were most important? Why?
- Can you predict what will happen?
- What was your estimate or prediction?
- How do you feel about your answer?
- What do you think comes next?
- What else would you like to know?

Equality and Equity Do all students participate to the same degree?

- Did you work together? In what way?
- Have you discussed this with your group? With others?
- Where would you go for help?
- How could you help another student without telling the answer?
- Did everybody get a fair chance to talk?

Solutions Do they consider other possibilities?

- Is that the only possible answer?
- How would you check the steps you have taken or your answer?
- Other than retracing your steps, how can you determine if your answers are appropriate?
- Is there anything you have overlooked?
- Is the solution reasonable, considering the context?
- How did you know you were done?

Extending Results Can students generalize their own answers? Do they connect the ideas to other similar problems or to the real world?

- What made you think that was what you should do?
- Is there a real-life situation where this could be used?
- Where else would this strategy be useful?
- What other problem does this seem to lead to?
- Is there a general rule?
- How were you sure your answer was right?
- How would your method work with other problems?
- What questions does this raise for you?

SIXTH GRADE GEOMETRY STRATEGIES

COMPETENCY GOAL 2: The learner will demonstrate an understanding and use properties and relationships of geometry.

2.1 Build models of 3-dimensional figures (prisms, pyramids, cones, and other solids); describe and record their properties

A. TETRAHEDRON: Build a tetrahedron out of a business size envelope that shows information about the student on each face such as student's name, a use for math in their everyday lives, their goal for math class, and a talent or hobby. Students should determine where on the envelope they will need to put their information before constructing the tetrahedron so that it can be seen.

Directions: Use a business size envelope. Seal the envelope. Draw diagonals and fold along the diagonals. Cut out the triangle at the top of the envelope flap area. Open up the remaining part. Tuck one half into the other half. Highlight the edges of each triangular side. The shape made is a tetrahedron. Unfold the shape to write the chosen information on each face

B. POLYHEDRA: Have students construct various types of polyhedra using drinking straws, string or yarn, and tissue paper. *See note. The students can explore the properties of faces, edges, and vertices by using the corresponding part of their model: tissue paper sides = faces, straws = edges, yarn intersections = vertices. Have students draw models of their constructions and complete a chart like the one below that will allow them to compare the polyhedra.

POLYHEDRA RECORDING CHART

Name of Polyhedra	# of Faces	# of Vertices	# of Edges

NOTE: Hair pins, available at beauty supply stores, can be used to join the straws in place of string or yarn. Place one hair pin in the end of one straw. Before inserting the second hair pin into the second straw, loop the pin through the first one. The hair pins are fast and quite useful for students who may have difficulty using the string.

C. EULER'S FORMULA: Using the chart completed in the activity above, ask students to analyze the number of faces, plus the number of vertices, minus two, equals number of edges. The analysis will lead to the discovery of Euler's formula: ($f + v - 2 = e$).

D. MODEL NETS: Have students create a net from a 3-dimensional model that either the teacher or a student has supplied. The students should cut out and build a replica. As an extension, students can write a description of their model in their journal or use the models and descriptions to create a bulletin board display. Nets can also be supplied for the students. **See Activity Sheet 2.1D.**

E. DODECAHEDRON: Using the dodecahedron pattern supplied students can construct a dodecahedron for a social studies report. Using pictures or information on each of its 12 sides, the students can represent different aspects of a country i.e., money, topography, imports, exports, government, symbols, etc. Especially this idea could be used for science reports, book reports, etc. As an extension of this activity, you may want to have students use a circle, and explore how the pattern for a dodecahedron is made.
See Activity Sheet 2.1E.

F. PENTOMINOS: Using graph paper and five different colored tiles students explore and discover all the different pentomino shapes. The tiles must be arranged so that each tile shares at least one side with another tile. Students try to find as many distinct arrangements as possible. There are 12 possibilities. Remind students that reflections and rotations of the same shape do not count as a new shape. Students should trace the pentominos on paper or color in squares on 1-inch graph paper to illustrate their findings. If they have difficulty determining if a shape has been reflected or rotated, cutting out the traced shapes will help them see if a new shape is distinct from others formed. After discovering and recording the pentomino shapes on graph paper, students explore to find out which pentominos can be folded to form a box without a top.

HEXOMINOS: Using the same procedures, students explore hexominos. Each student or group works with six tiles. Once they have found and recorded as many shapes as possible, students predict if each arrangement can be folded to form a cube. There are 35 possibilities. Students cut out and fold each design to test each prediction. Students then discuss the similarities observed in the arrangements that formed cubes.

G. PACKAGE DESIGN: Give students different irregularly shaped objects such as a styrofoam cup, a tube of toothpaste, or a pine cone. In teams they design a package with adequate volume to hold their object, and the least possible surface area to minimize the amount of material needed to make the package. The dimensions of the larger container should be supplied to the students and the students should be encouraged to make 3-dimensional models that will demonstrate the effectiveness of their solution.

H. POLYHEDRAS: Using gumdrops or marshmallows with toothpicks, students construct the structural outlines for different polyhedra. Students can eat their creations after they have described and recorded the name of polyhedron, number of faces, number of edges, and the number of vertices. They should use Euler's formula to check their results.

I. GEO-BLOCKS: Using various geo-blocks, students trace all the faces of a single geo-block on card stock and cut them out to create a puzzle. Another student identifies the geo-blocks of the puzzle by looking at the pieces. Next, students trace the individual puzzle pieces to design the net which would fold to create the original block. The students can then cut out the net and construct a duplicate of the model, or block to verify their solution.

J. POLYHEDRAS: By rolling and tracing various geo-blocks, students can create nets, cut them out, and construct duplicate models of the polyhedra. Students write descriptions, sort, and classify their models in different ways such as number of sides, number of edges, type of polyhedra, etc. Students use the "Polyhedra Sort" Venn Diagram to report their results. Alternately, the teacher can duplicate this Venn diagram in large scale on bulletin board paper and lay it on the floor. Students then place their models in the appropriate place on the diagram. **See Activity Sheet 2.1J.**

K. CITY SKYLINE: Give students a picture postcard or a 2-dimensional picture of a city skyline and have them use polyhedra to construct a 3-dimensional model of the city as seen from the same perspective. **See Activity Sheet 2.1K.**

L. Using orthographic drawings of only three sides of a 3-dimensional figure, have students identify the figure, its number of edges and faces, and the shape of its base. Students draw the figure isometrically. See examples of orthographic and isometric projections in **Activity Sheet 2.1L.**

M. ORTHOGRAPHIC DRAWING: From an orthographic drawings of a 3-dimensional model or a building, students use geo-blocks, unifix cubes, or Cuisenaire rods to construct the model. **See Activity Sheet 2.1M.**

N. THEME CUBES AND TETRAHEDRONS: Arrange students in groups of five or six. Distribute the student rubric and cube or tetrahedron nets. Each group should pick a common theme for their cube or tetrahedron figure. Each student decorates one side of the figure using a design that relates to the group's theme. Next, each student cuts out a side and the group will construct their cube or tetrahedron. Groups should then complete the rubric and answer the processing questions. The figures can be displayed by punching a hole in any vertex then hanging them with string or an unfolded paper clip.
See Activity Sheet 2.1N.

O. HUNT FOR GEOMETRIC SHAPES: Divide students into groups and send them to different locations to find geometric shapes in the environment. In groups, students make a graph to display the shapes found. Compile class results. Discuss which shapes are most predominant and why.

2.2 Classify angles (interior, exterior, complementary, supplementary) and pairs of lines including skew lines.

A. GEOMETRIC SHAPE CONSTRUCTION: Have students construct various geometric shapes when the measurements of some or all of the interior angles are given. Have students describe and record the properties of the shapes they have created such as name of shape, number of sides, number of angles, the sums of all interior angles, similarities, and differences of shapes.

Example: Given three angles which measure 38°, 52°, and 70°, construct a pentagon. Describe the process you used to arrive at an answer; make a drawing and record a description of your pentagon.

B. CONVEX AND CONCAVE POLYGONS: Students use geoboards to construct various convex and concave polygons to explore the concept of interior and exterior angles. Using geoboard paper or their math journals, students draw the polygons, color code the angles according to their classification, and define "interior" and "exterior" as the terms related to angles of polygons.

C. ANGLES: Give students an empty square and have them use only straight lines to create a drawing. Have students identify the various types of angles formed in their drawing such as acute, obtuse, right, complementary, supplementary. Students can color-code their drawings to represent the types of angles.

D. STIR CRAZY GEOMETRY: Using coffee stir sticks, pencil, and paper, students explore and discover all the possible ways that lines (not line segments) intersect in a plane. They record their answers on the given chart. **See Activity Sheet 2.2E.**

E. ANGLES: Using different pictures of lines (intersecting, parallel or perpendicular lines in a plane), students explore the concept of complementary and supplementary angles, interior and exterior angles, and types of angles formed by various lines. Relate this to "Stir Crazy Geometry." Students could color in all pairs of complementary or supplementary angles. **See Activity Sheet 2.2F.**

F. INTERIOR ANGLES: Students use various regular polygons such as pattern block shapes to explore the sum of the measurements of interior angles in each polygon. Rather than having them measure the interior angles with a protractor, have them explore the interior angles by drawing diagonals from one vertex to all other vertices in the polygon, thus splitting" the polygon into triangles. Students can draw the shapes and then describe and record their findings about each shape using a chart like the one on the following page that identifies the number of sides, diagonals, and triangles created within the shape, and total measurement of all interior angles. By drawing diagonals from only one vertex, students may discover: number of sides of polygon minus two equals the number of triangles formed, and the number of triangles times 180° equals total sum of all the interior angles.

POLYGON RECORDING CHART

Name of polygon	number of sides	number of diagonals	number of triangles	Total number of degrees

G. OBSERVE AND DISCUSS: Observation - Choose an everyday item to examine with students. The item should be large enough for class observation such as a bike, Tonka truck, or battery powered car. Examine the lines used in the design to determine which geometric elements have been utilized in this product. Students might identify types of angles including supplementary and complementary pairs, parallel and perpendicular line segments, and various polygons used in the design of the item.

Discussion - How do these features affect the overall appearance of the product?

Extension - How could you change the appearance of this item by changing one or more of its geometric elements? How could you make the item appear more streamlined, larger in size, stronger? Evaluate these changes. Would each be an improvement? Justify your answer.

H. MARSHMALLOW MODELS: Show children models of geometric shapes. Distribute miniature marshmallows and toothpicks for children to create their own models. Dip the shapes into soapy water. Examine the models for angles and lines made by the soap bubbles. Look for interior angles, obtuse and acute angles, and the different line segments that become apparent.

I. CITY MAPS: Use city maps or draw a map of the streets in your town. Look for parallel lines and perpendicular lines in the street layout. Estimate the size of the angles formed by the intersections of the streets. Discuss why parallel and perpendicular lines may be used in the design of the city streets. How can this aid navigation? "Jenny's Journal", a computer game by MECC, focuses on map skills and following directions. The map given in "Jenny's Journal" can be used to discuss the use of parallel and perpendicular lines in following location directions.

J. GEOMETRY ON THE GO: As a warm-up activity, take students on a walking tour of your school campus. Be sure they take along a pencil, paper, and a notebook. Identify geometric elements such as angles, types of lines, geometric solids, etc. observed. Students might locate these elements by examining school buildings, landscaped areas, or recreational fields/areas. Visit the parking lot. Have each student select a vehicle to sketch. Students should draw several views of the selected vehicles. Example: front, side, back. Students sketch and label the geometric elements of that vehicle. For example: supplementary angles are found where the windshield, hood, and dashboard meet. Once drawings are completed, have students compare and discuss the sketches in pairs or small groups. What elements are similar? What elements are different? How do the angles located in the body of a pick-up truck differ from those found in a sports car? Display the drawings.

K. Use pattern blocks to make examples of drawings with lines of symmetry. Divide the class into groups and have a competition to see which group can use the pattern blocks to make the largest figure with one or two lines of symmetry.

L. Using the lesson "Symmetry Coloring" from Mathematical Toolbox students color designs according to directions. **(See Activity Sheet 2.2 L.)**

2.3 Construct congruent segments and congruent angles. Construct bisectors of line segments using a straight edge and compass.

A. Have students construct congruent segments and angles on tracing paper. After completing their drawings, they can overlay the tracing paper on the original to verify their accuracy.

B. Using a straight edge and a compass students construct bisectors of various line segments. Students can fold the paper on the bisector to verify their accuracy or they can fold the paper by matching up the two end points.

C. LIFE-SIZED CONSTRUCTIONS: Working in pairs or larger groups, students place masking tape on the floor to create an original line segment. (Marking a line on the masking tape, rather than using the ends of the tape, will make the construction more accurate. The groups place another piece of masking tape down on the floor, longer than the first. Using string and a pencil or fine-line marker, pairs work through the construction of a congruent line segment just as they would on paper. One group member would hold the end of the string on "point A," another group member would extend the string to "point B," etc. This kinesthetic activity could be extended with other constructions as well, and will certainly give students the "big picture!" Discuss with students the possible loss of accuracy with this method. Extend discussion to include the idea that a sharpened pencil and minuscule points on a pencil-and-paper construction make the drawing much more accurate. In other words precision of the construction depends upon the precision of the instruments.

D. QUILT DESIGNS: Show students examples of quilt square patterns and discuss geometric designs using constructions whenever possible. Students create an original quilt design on an 8 x 8 square of white drawing paper, using constructions whenever possible. They may use any or all of the following geometric concepts: symmetry, translation, reflection, rotation, congruency, similarity, etc. Add color on their journal and complete a written explanation of the geometric features of their design. Students can exchange designs and write an explanation of their partner's design; only display explanation of pattern on a bulletin board. Each student will then choose a geometric pattern from the bulletin board and try to duplicate it by following the description. Connect all class designs to create a class quilt. Display the quilt.

E. Arrange students into cooperative learning pairs. Using a pencil and straight edge the first student draws parallel lines. The second student draws perpendicular lines. Students exchange papers and construct the other person's drawing using a pencil and straight edge.

2.4 Identify and distinguish among similar, congruent and symmetric figures; name corresponding parts.

A. SYMMETRICAL REFLECTIONS: Show students one-half of a letter, picture, number, shape, or design, and have them create the symmetrical half or reflection. Students who are experiencing difficulty with this task may find it helpful to use a mirror placed on the vertical or horizontal axis to see the reflection. They should label three major points on the original design and label the corresponding point on the reflection.

B. NATURE SYMMETRY: Go on a nature walk. Collect and record various symmetrical objects, such as leaves, pine cones, and butterfly wings. Have students draw the objects and mark the lines of symmetry for each item.

C. SNOWFLAKE SYMMETRY: — Have students explore symmetry and lines of symmetry by making their own snowflakes.

Directions:

1. Begin with a square sheet of paper.
2. Fold the paper in half vertically.
3. Fold in half again horizontally.
4. Fold in half again along the diagonal which bisects the folded corner.
5. Cut designs along the two folded edges. The more cuts that you make, the more intricate your design will be.
6. Carefully unfold your symmetrical design.
7. Identify all lines of symmetry. Place the design between or under plastic and draw these lines with an overhead projector pen to preserve the beauty of your snowflake.

D. FOOD SYMMETRY: Bring in a variety of fruits and vegetables for students to use in exploring the concept of symmetry. Have students predict where the different lines of symmetry would be when they are cut. Students can cut the vegetables along those lines to test their predictions. As an extension, create fruit and vegetable prints, patterns or borders using finger paint and slices of fruits and vegetables.

E. LETTER AND NUMBER SYMMETRY: Choose a letter or number and ask the class to show the line(s) of symmetry. Some students may find it necessary to use a mirror to complete this task. After determining which letters and numbers are symmetrical, the teacher can challenge students to use these to create a phrase, sentence, larger number, or equation thatwould also be symmetrical.

F. NAME SYMMETRY: Have students print or write their names on the fold of a sheet of paper, then flip the paper over and trace their names. Unfold to show the reflection or mirror image. Color, cut, and mount as desired.

G. PATTERN BLOCK SYMMETRY: Using examples of drawings created from pattern block tracings, cut and fold the drawings to discuss the concept of symmetry. Students can then use pattern blocks to draw a pattern of their own and show the lines of symmetry using string, coffee stir sticks, spaghetti, or pick-up sticks. As an extension, have students work in pairs to create pattern block reflections. Students put down a representation of a line of symmetry (piece of yarn, spaghetti, etc.). The first student places a pattern block on his/her side of the line; the second student creates the mirror image (reflection) of the first piece. The two students continue taking turns building a design and must then place the same pattern block on the other side of the line to design its reflection. These designs can be traced onto paper or display purposes.

H. CONGRUENT FIGURES: Give students any geometric figure with its points labeled, and have them construct a congruent figure using a straight edge and compass. Students should label their points with different letters and identify the corresponding parts of both figures.

I. CONGRUENT OBJECTS: Look in the classroom or around the school to find similar and congruent objects or items with lines of symmetry. Make sketches of the objects and label them as similar or congruent. Draw any lines of symmetry.

J. FLAG SYMMETRY: Using any reference materials, students find a flag that has at least two lines of symmetry. Students sketch the flag and identify its lines of symmetry.

K. GEOBOARD CONSTRUCTION: Using an overhead geoboard and rubber band(s), the teacher makes a figure. Have the students make a congruent and/or a similar one on their own geoboards.

L. GEOBOARD CONSTRUCTION: Using an overhead geoboard and rubber band(s), the teacher identifies a line of symmetry and designs one half of a figure. Have students replicate the line of symmetry and the second half of the figure.

2.5 Recognize the results of translations, reflections, and rotations using technology when appropriate.

A. QUILTING PATTERNS: Give students a quilting pattern and the elements used to create the pattern. Ask students to recreate the pattern. Complete a "quilted" sheet. Students may use translations, reflections, or rotations as needed. As an extension, students could then make a pattern themselves, and identify the elements needed to construct it. **See Activity Sheet 2.5 A.**

B. BORDER CONSTRUCTION: Using a unit design, students create a border using translations, reflections, and rotations of the chosen design. Students can investigate uses of borders in artwork, manuscript decoration, architecture, and other areas of graphic design. Students can also bring in examples and classify the types of transformations that created the borders.

C. PATTERN BLOCK DESIGNS: Have students explore pattern blocks and pattern block designs and then create new patterns that involve translations, reflections, and/or rotations. After manipulating the blocks, students can record their creations, add to or change their design using other colors, and then mount and display them.

D. COMPUTER GENERATED DESIGN: Use the computer program LOGO Writer or some other version of the computer language LOGO to provide a shape on the computer. Have students explore and use LOGO commands to create a congruent shape or a symmetrical image. Students can also investigate whether they can flip, rotate, or slide the shape. After experimenting with writing their own commands to change the original shape, give the students the original program statements and see if they can create a new program or rewrite an existing program to do these things with other shapes.

E. Using letters of the alphabet, show the rotation and reflection of each. Students can create mystery sentences using transformed words.

F. Using their body, students demonstrate translations, reflections, and rotations.

2.6. Explore changes in shape through stretching, shrinking, and twisting.

A. BALLOON PLAY: Have students write messages on both deflated and inflated balloons. You can sometimes get balloons from local banks or stores that have pre-printed messages. Students explore stretching, shrinking, and twisting by inflating and deflating the balloons several times. The long balloons used to make animal figures and other shapes are especially good for observing twisting.

B. RUBBER BAND PLAY: Students stretch a wide rubber band across a book and write a message on the rubber band with a permanent magic marker. Students record their observations both before and after they remove the band from the book. If the rubber band has a personal message on it (e.g., for Valentine's Day), it can be given to the person for whom the message is intended. Wide rubber bands can sometimes be obtained in the produce section of your grocery store. Ask parents to save them for you throughout the year!

C. LATEX GLOVE PLAY: Using latex gloves, students draw geometric shapes on them and then stretch the gloves in various ways to observe and record the changes that occur in the original shape.

D. COMPUTER GENERATED CONSTRUCTION: Using the LOGO computer language, students take a shape provided by he teacher on the computer and shrink or stretch it by rewriting the program to make the shape larger or smaller.

E. COMPUTER SOFTWARE EXPLORATION: Using various commercially prepared computer software, have students explore how these different software programs stretch or shrink letters, shapes, or figures. For example, any word processing program that can vary font size can be used. Students type in a message, then change the font size to observe the difference in the appearance of the letters. If the "outline" format is available, this is a useful tool for observing how the letters change.

F. SHRINKING AND TWISTING EXPLORATION: If available at your local arts and crafts store, purchase a few "Shrink Art" or "Shrinky-dink" packages to help your students explore the concepts of shrinking and twisting. Beginning with the special plastic, a design is drawn onto the plastic, cut out, and then heated in an oven. The plastic will shrink to a smaller size and may also twist slightly in the process creating a much smaller image that may sometimes be slightly distorted due to the twisting.

G. PEEP HOLE: Challenge the students to cut a piece of notebook paper to create a hole that they can step through. They may make as many cuts as they wish, but only 1 piece of paper and scissors can be used (no glue, tape, etc.) For the solution, read " Peep Hole." **See Activity Sheet 2.6 G.**

2.7. Recognize geometry in the environment (e.g. art, nature, architecture).

A. NATURE GEOMETRY: Have students go on a nature walk or a walking field trip into the city to collect and/or record various geometric objects or figures in the environment. Students can later discuss and classify these by types of lines, shapes, symmetrical objects, congruent objects, etc.

B. TETRAHEDRA RESEARCH AND CONSTRUCTION: Have students research Alexander Graham Bell and his work with tetrahedral kites. Using the information they have collected, prior knowledge with tetrahedrons, and other polyhedra, students construct models of different tretrahedral or polyhedral kites. Students test them by actually trying to fly them.

C. GEOMETRY HOMEWORK: As a homework assignment, ask students to notice some geometric figure on their way home. They should either draw the figure or, if possible, bring it in to demonstrate and/or describe what geometric properties the figure possesses.

D. CAREER DEVELOPMENT: Invite a landscape architect, interior designer, or building architect to visit your class and share information concerning current technology or software products that are used to create 3-dimensional designs on paper. They may be able to demonstrate this in class or your class may be able to take a field trip to their office. They may also be able to explain details about how these designs are translated into the "real" thing, how math plays a part in their job field, and how geometry, in particular, is applicable to their job duties.

E. CAREER DEVELOPMENT: Invite a graphic design artist or printer to visit your class and share information about his/her job, the technology used to produce a product, and how math (geometry in particular) is used in his/her job field.

F. BORDER EXPLORATION: Have students find borders in magazines, advertisements, posters, CD covers, etc. that show symmetry, reflections, rotations, translations, stretching, shrinking, and twisting, to share and discuss with the class.

G. DRAWING: Have students draw their own building using as many geometric shapes as they can. They should show lines of symmetry, identify the shapes of polygons used, and identify skew, parallel and perpendicular lines.

H. DESIGN EXPLORATION: Using a quilt, posters for advertisements, or jackets for CD's and records, discuss the geometric elements in design that make it appealing. Students can bring examples from home to share.

I. OBSERVING AND BUILDING BRIDGES: This activity works well with small cooperative groups and should take about five class periods. Science, communication skills, and math are integrated.

1. OBSERVATION: First, students observe different types of bridge constructions. Discuss the geometric elements observed in each such as intersecting lines, parallel lines, lines of symmetry, polygons, cylinders, repeated congruent shapes, etc. After discussing observations, explain the investigation.

2. DESIGN: Students use ten sheets of duplicating paper, scissors, and scotch tape to design and build the tallest possible bridge that will support a brick. The tape can only be used as an adhesive, not as a strengthener. After giving directions, students are put into cooperative groups and asked to respond to this problem in their math journals by completing the following statements. (This is what I've been asked to do....This is our strategy....This is what we hope it will look like...", etc.).

3. BUILD: During the next class period, ask students to share their journal entries with their group to check for understanding of the task. Distribute materials and let groups begin work. At the end of the period, ask students to reflect on their work: "Has everyone been involved?" "Are things going well?" "Do you need to re-think your design plan or start again?" "Can you make any predictions of what might happen?" Allow students time to complete constructions. Let the class predict which bridges will support the brick—then test. Compare constructions, noting geometric designs. Students write evaluation in journals: "Describe the design used in the bridge(s) that supported the weight of the brick." "Compare this design to your bridge design if it is different."

L. GEOMETRY IN THE FUTURE: Have students make a futuristic design such as pencil, car, or vacuum cleaner using polygons; parallel lines; perpendicular lines; acute, obtuse, and right angles; lines of symmetry; skew lines; intersecting lines; supplementary and complementary angles; and any other geometric concept they have learned thus far. Students should utilize their knowledge of these concepts by describing their design well enough so that their written description and a sketch of their design can be matched. Students should name their design, describe its purpose, and explain why the design will be helpful in the future.

Additional Activities

A. OVERHEAD CONCENTRATION: Using the "Overhead Concentration" sheets provided, make a transparency and play "Overhead Concentration" to help students build a greater understanding of terminology and recognize different geometric figures. Small post-it notes can be used to cover the answer squares. They can be removed to uncover an answer and then be re-used to cover any incorrect choices made by the students. **See Activity Sheet 2.0A.**

B. OVERHEAD CONCENTRATION: Give students copies of the "Overhead Concentration" master and have them create their own "Concentration" game boards. The teacher can make an overhead transparency to allow the entire class to play, or students can exchange papers among themselves and play in pairs or small groups. **See Activity 2.0 B.**

C. GEOMETRIC SCAVENGER HUNT: Have students go on a scavenger hunt to locate a list of items with specified geometric properties. Items could include, but are not limited to:

- something that shows horizontal symmetry
- something that shows vertical symmetry
- something that shows radial (multiple) symmetry
- something with 3 or more acute angles
- something with 7 or more congruent angles; etc.

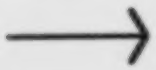
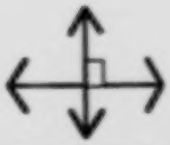
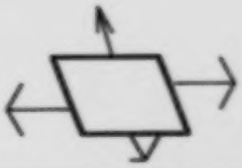
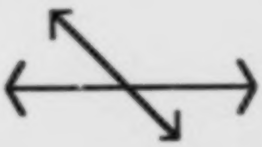
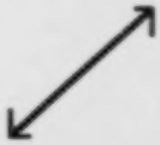
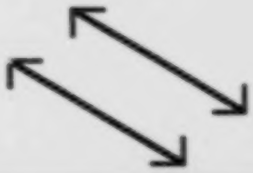
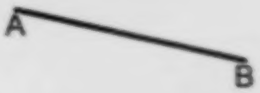
D. GEOMETRIC SHAPES: Given various geometric shapes, students construct congruent and/or similar shapes. Name the geometric shapes and the types of angles in the figures. Students construct diagonals or draw lines of symmetry.

BLACKLINE MASTERS

SIXTH GRADE

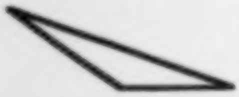
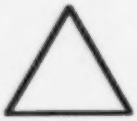
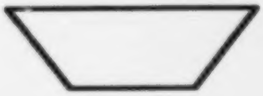
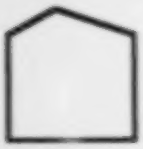
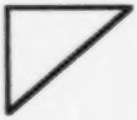
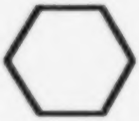
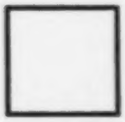
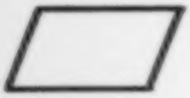
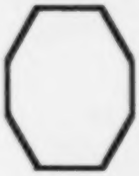
STRATEGIES

OVERHEAD CONCENTRATION

1 parallel lines	2 line segment	3 ray
4 	5 intersecting lines	6 
7 line	8 	9 
10 	11 	12 skew lines
13 plane	14 	15 point
16 perpendicular lines	17 non-parallel lines in space which never intersect	18 

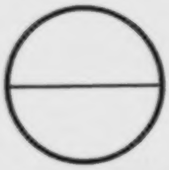
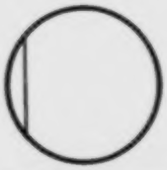
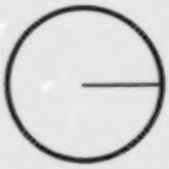
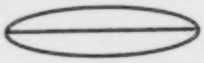
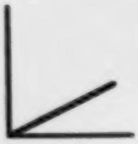
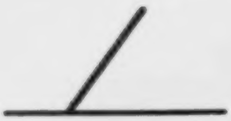
Activity Sheet 2.0 A & B

OVERHEAD CONCENTRATION

1 	2 trapezoid	3 isosceles right triangle
4 octagon	5 	6 
7 	8 	9 pentagon
10 equilateral acute triangle	11 parallelogram	12 hexagon
13 scalene obtuse triangle	14 	15 
16 	17 square	18 

Activity Sheet 2.0 A & B

OVERHEAD CONCENTRATION

1 radius	2 180°	3 circumference
4 chord	5 	6 
7 3.14 or $\frac{22}{7}$	8 	9 # of degrees in a straight line
10 	11 arc	12 π
13 diameter	14 the distance around a circle	15 
16 complementary angles	17 	18 supplementary angles

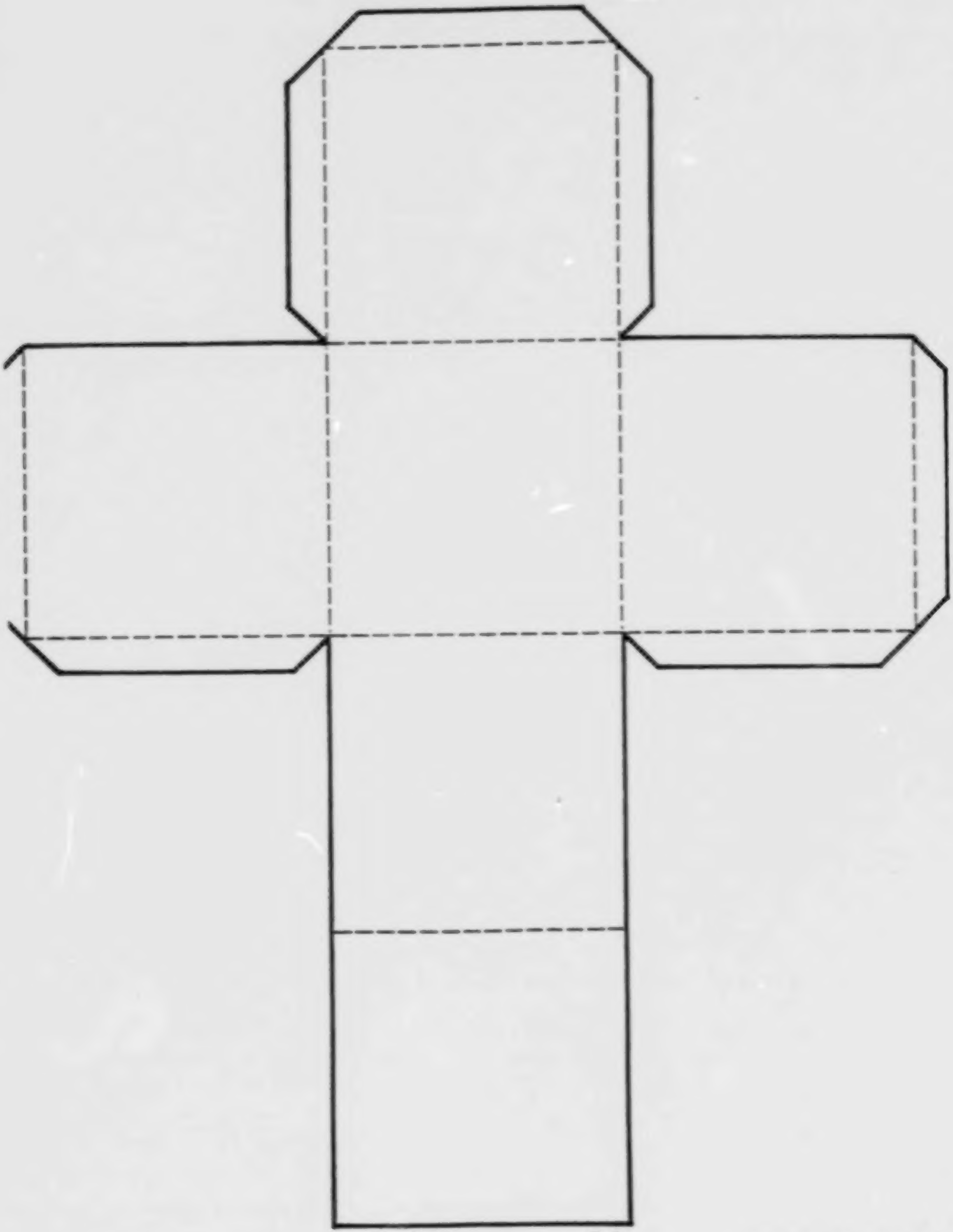
Activity Sheet 2.0 A & B

OVERHEAD CONCENTRATION

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18

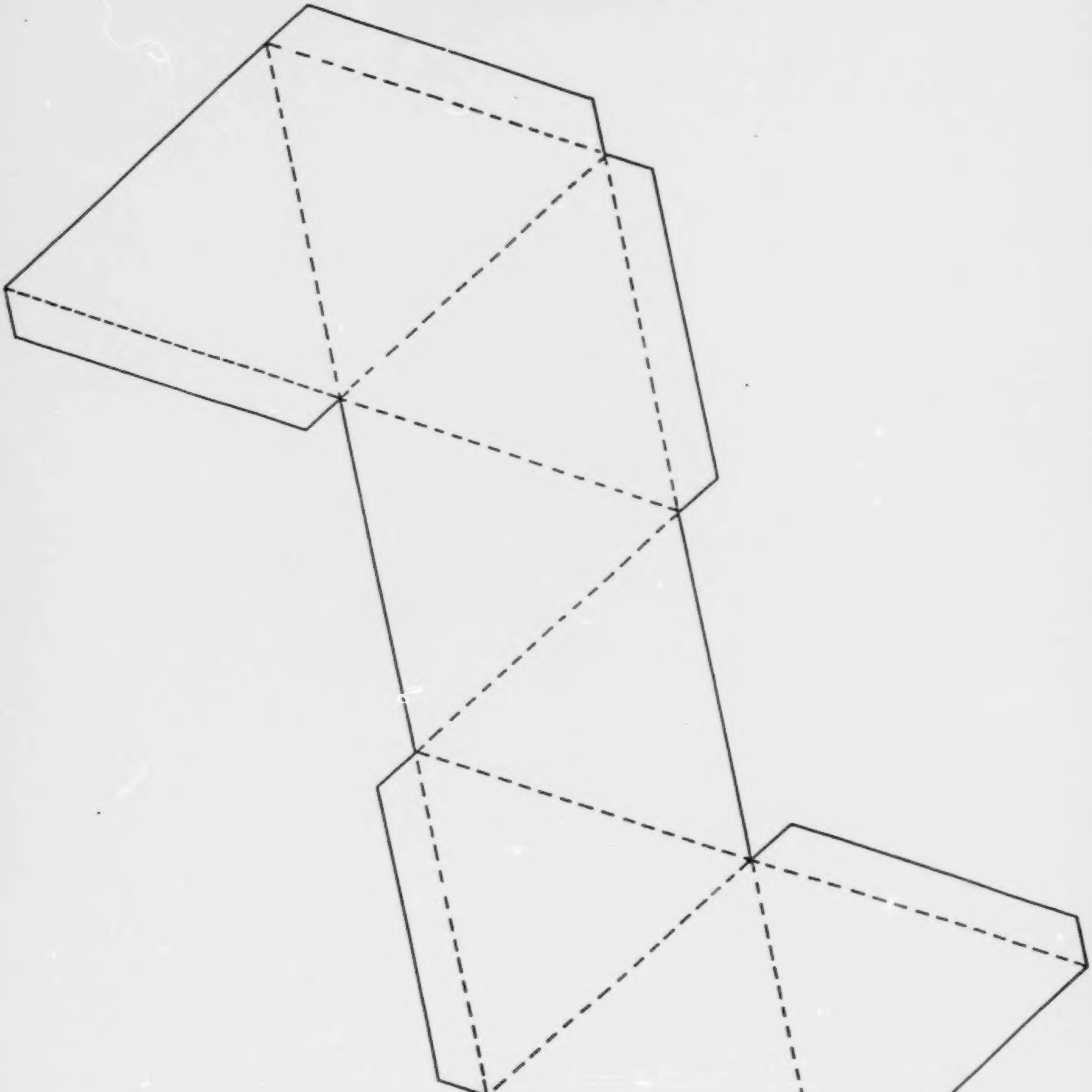
Activity Sheet 2.0 A & B

HEXAHEDRON NET



Activity Sheet 2.1 D

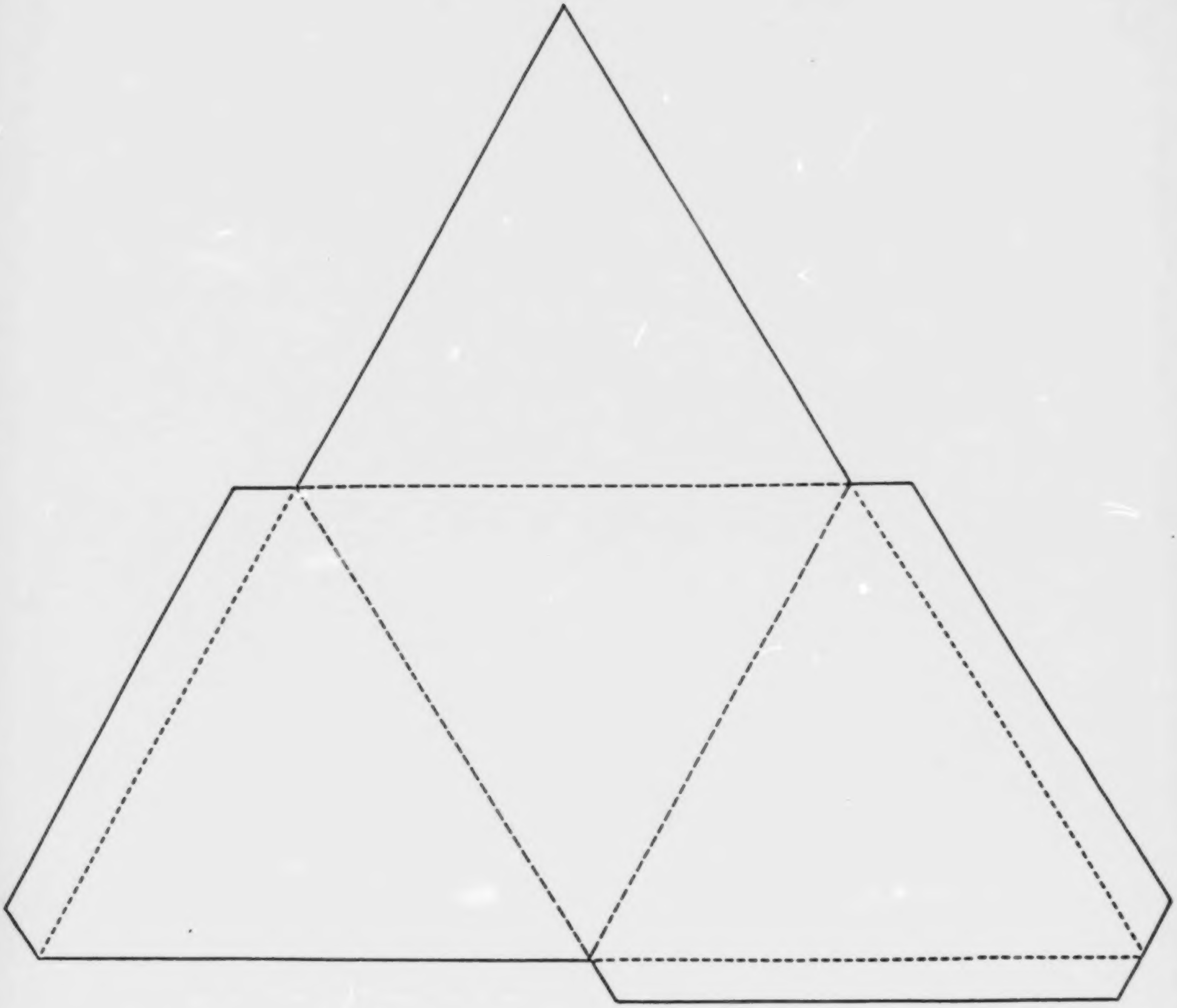
OCTAHEDRON NET



Activity Sheet 2.1 D

34

TETRAHEDRON NET



Activity Sheet 2.1 D

DODECAHEDRON NET

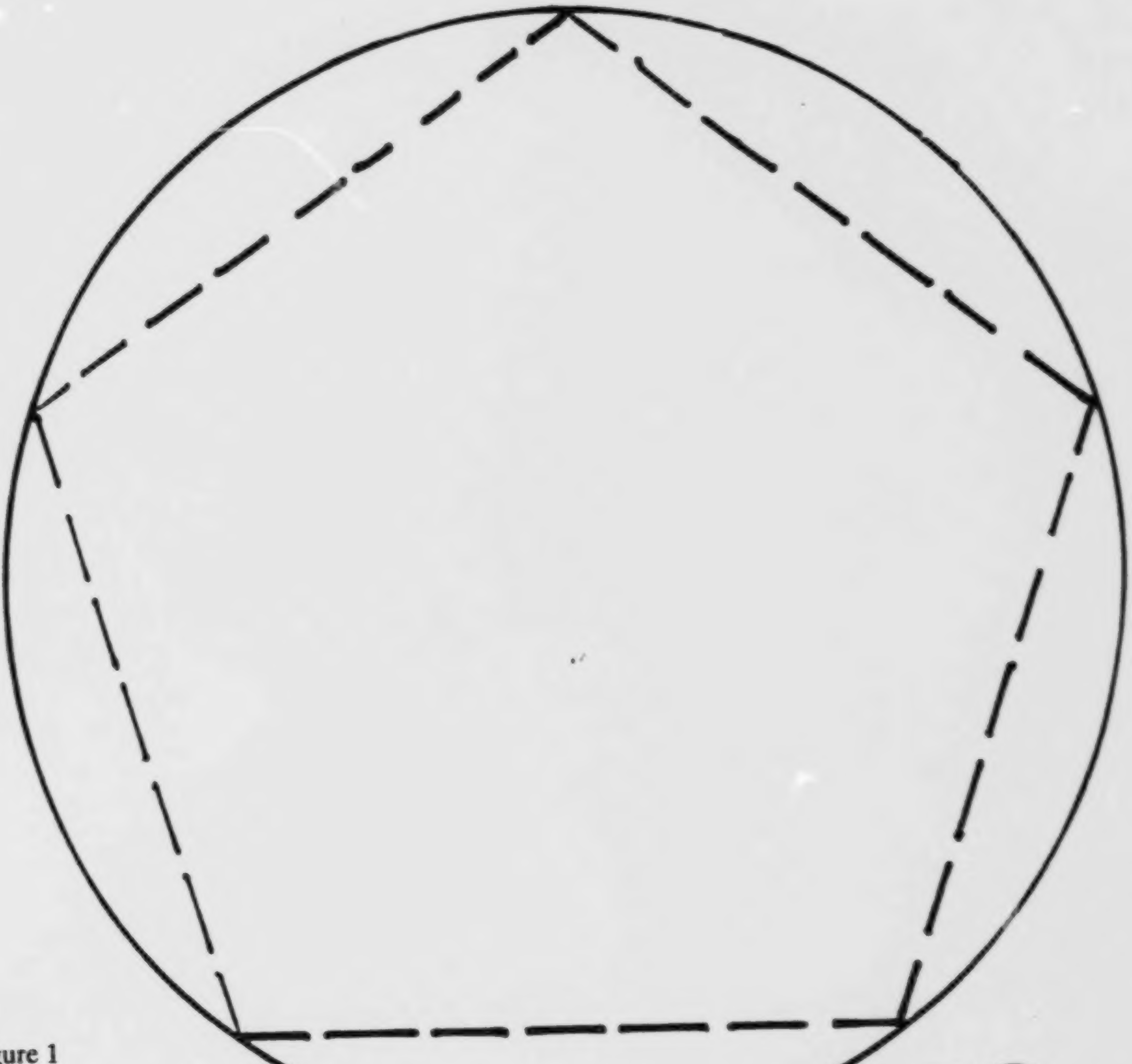


Figure 1

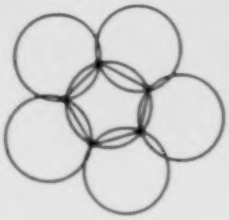
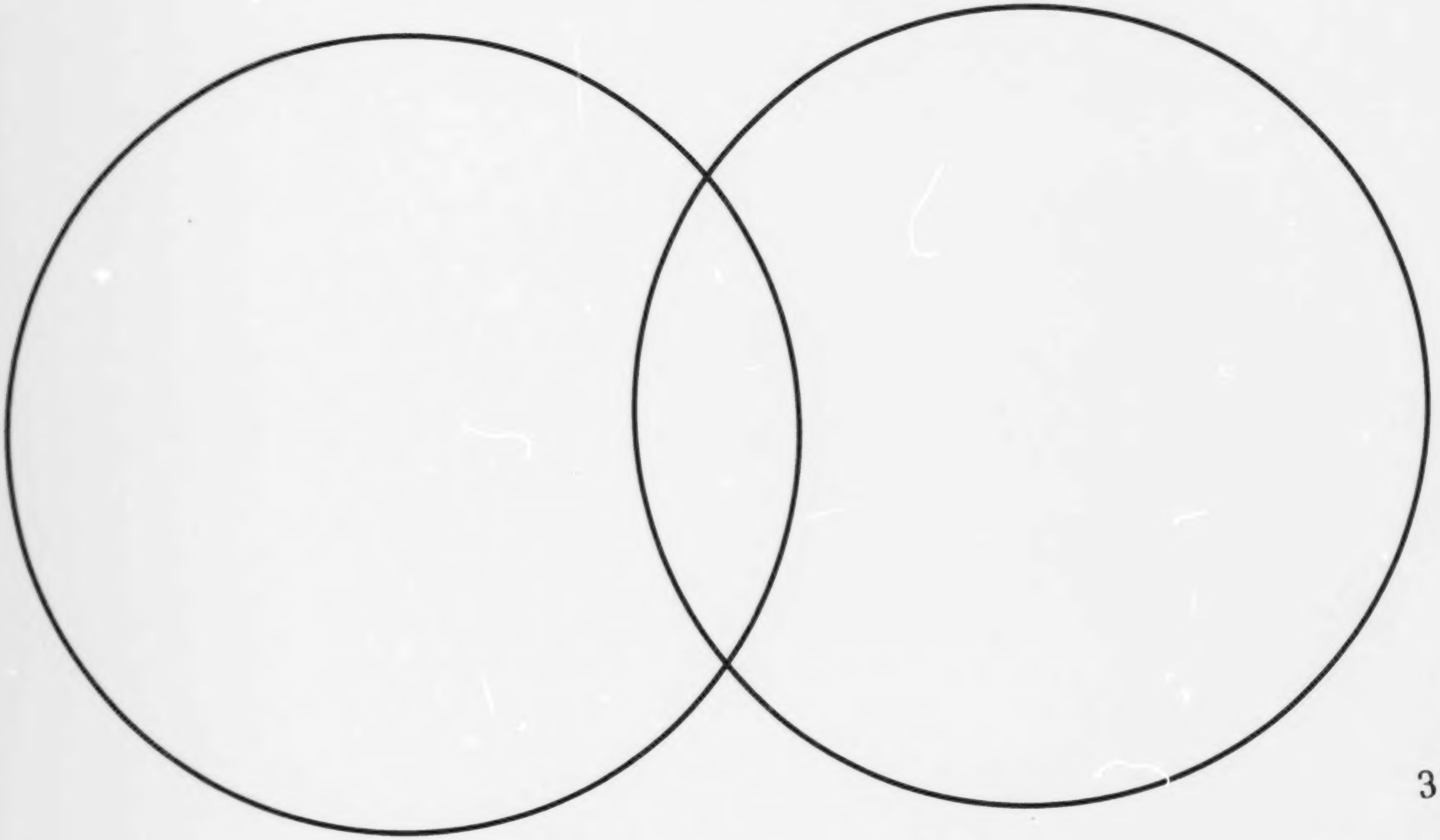


Figure 2

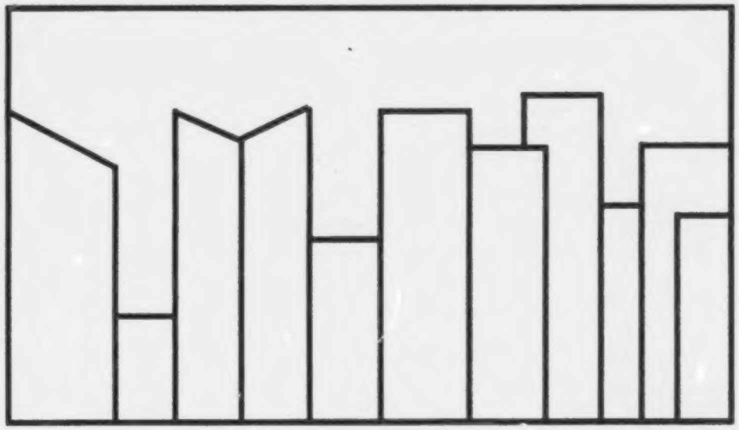
Cut out 12 patterns (Fig. 1). Fold tabs toward center of circle on dotted lines on all 12 patterns. Create two "flowers" by stapling 6 patterns together as shown in (Fig. 2). Be sure tabs are folded to the outside. Then, holding the 2 "flowers" together, staple all matching tabs together while rotating them.

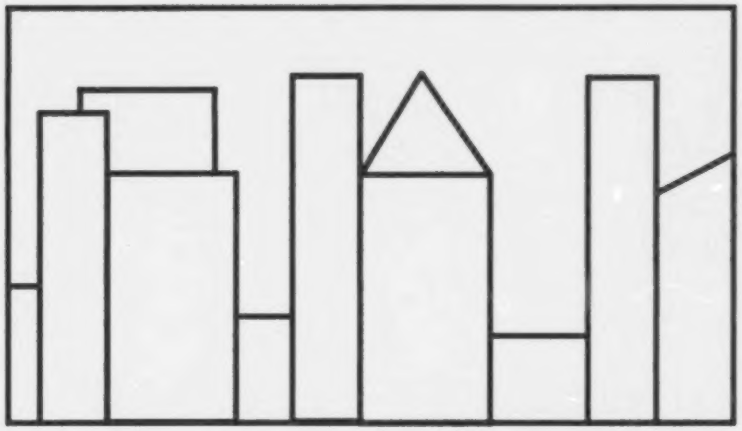
Activity Sheet 2.1 E

POLYHEDRA SORT

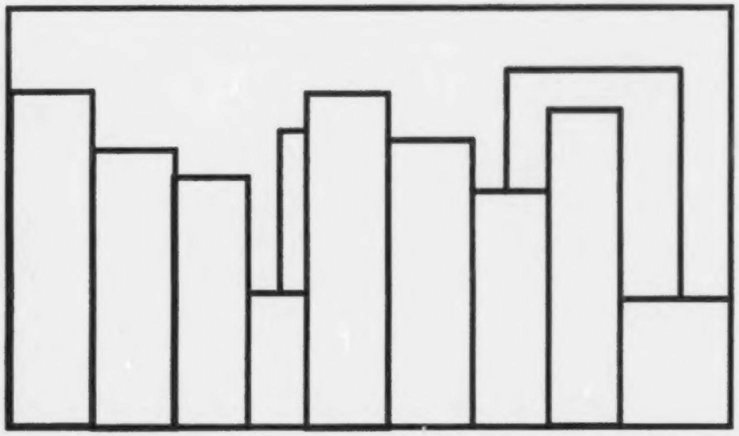


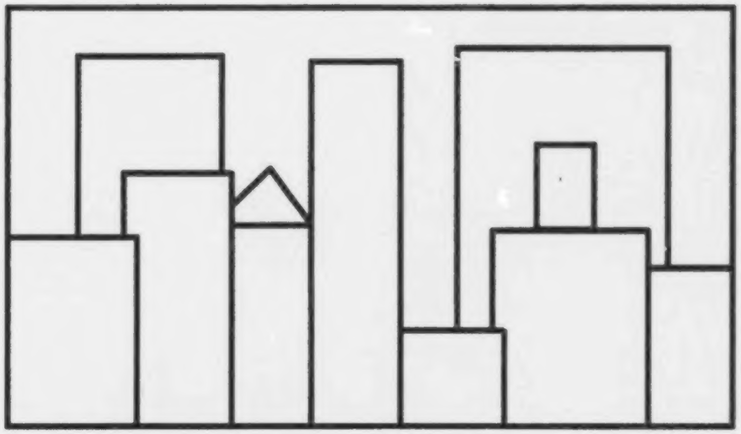
Activity Sheet 2.1 J





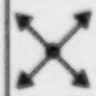
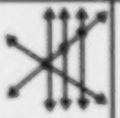
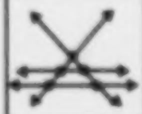
CITY SKYLINES





Activity Sheet 2.1K

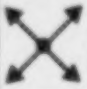
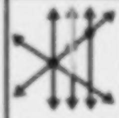
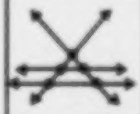
STIR CRAZY CHART

NUMBER OF LINES	NUMBER OF INTERSECTING POINTS						
	0	1	2	3	4	5	6
2			NOT POSSIBLE	NOT POSSIBLE	NOT POSSIBLE	NOT POSSIBLE	
3							
4							
5							
6							

Activity Sheet 2.2 E

STIR CRAZY CHART

NUMBER OF INTERSECTING POINTS

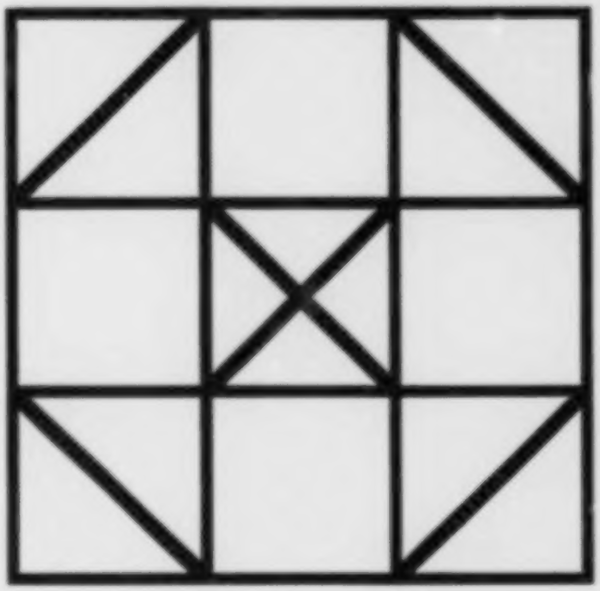
NUMBER OF LINES	0	1	2	3	4	5	6
2			NOT POSSIBLE	NOT POSSIBLE	NOT POSSIBLE	NOT POSSIBLE	
3							
4							
5							
6							

Activity Sheet 2.2 F

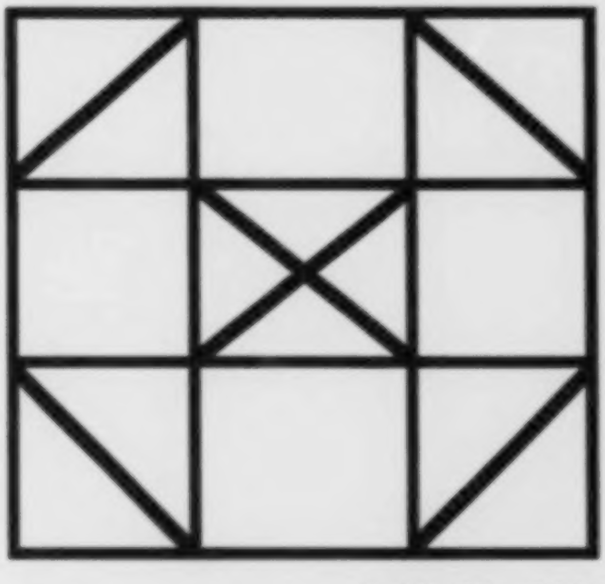
Symmetry Coloring

Adapted from Mathematical Toolbox, Challenge Set 31, Activity D
Color each design following these directions:

- Using exactly 1 line of symmetry



- Using exactly 2 lines of symmetry

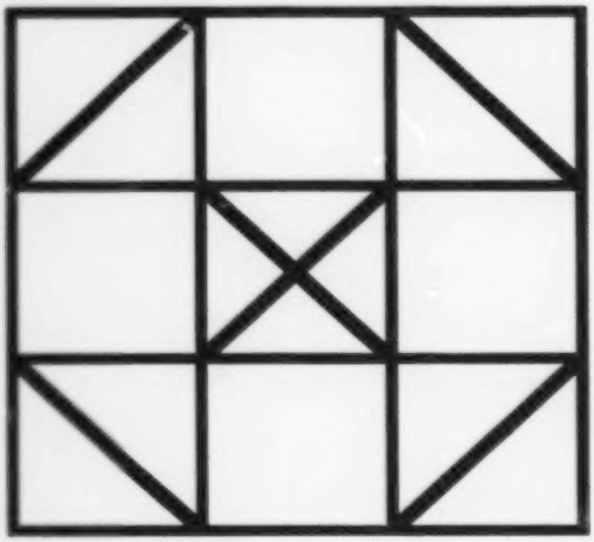


Activity Sheet 2.2 L

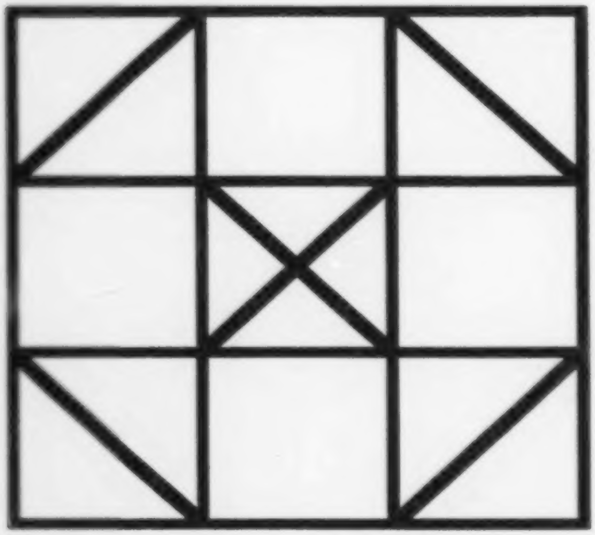
Symmetry Coloring

Adapted from Mathematical Toolbox, Challenge Set 31, Activity D
Color each design following these directions:

- Using exactly 4 lines of symmetry

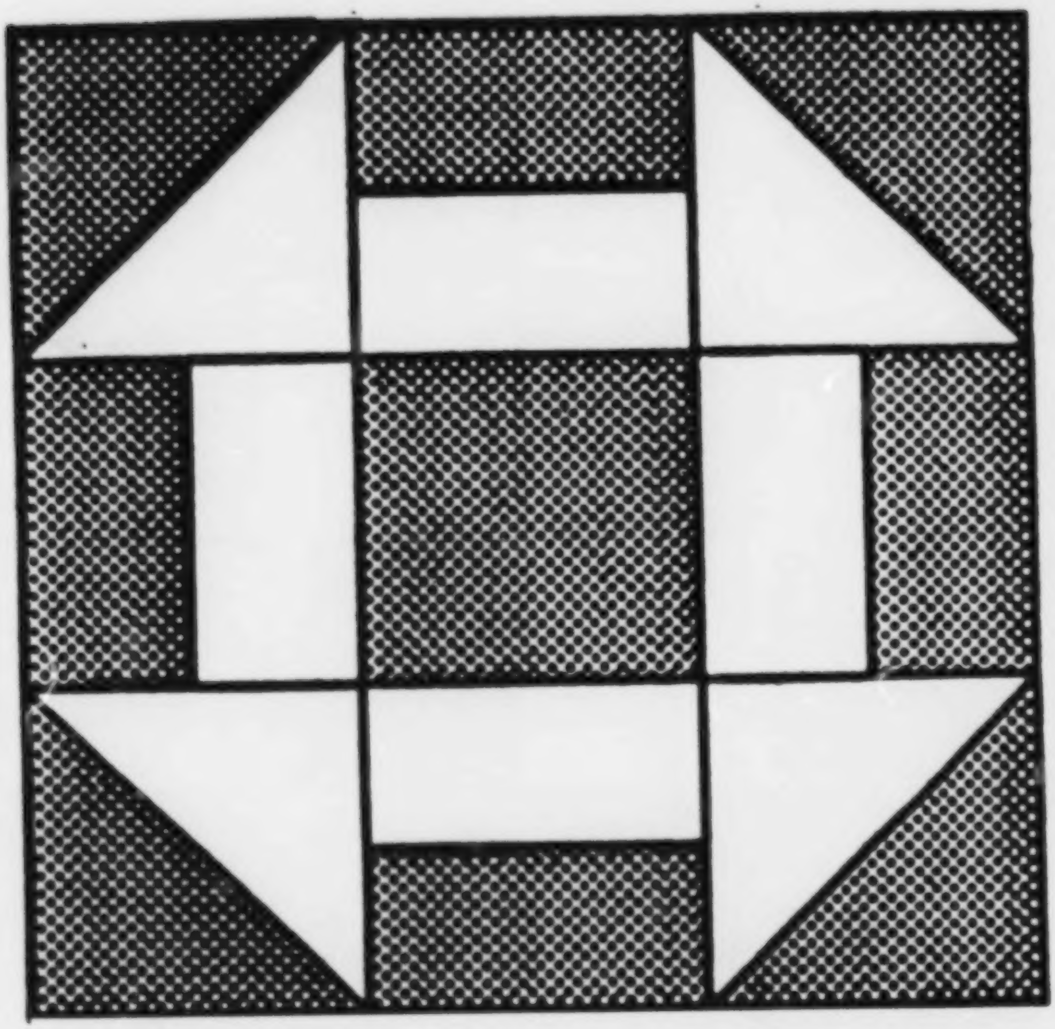


- Using no lines of symmetry



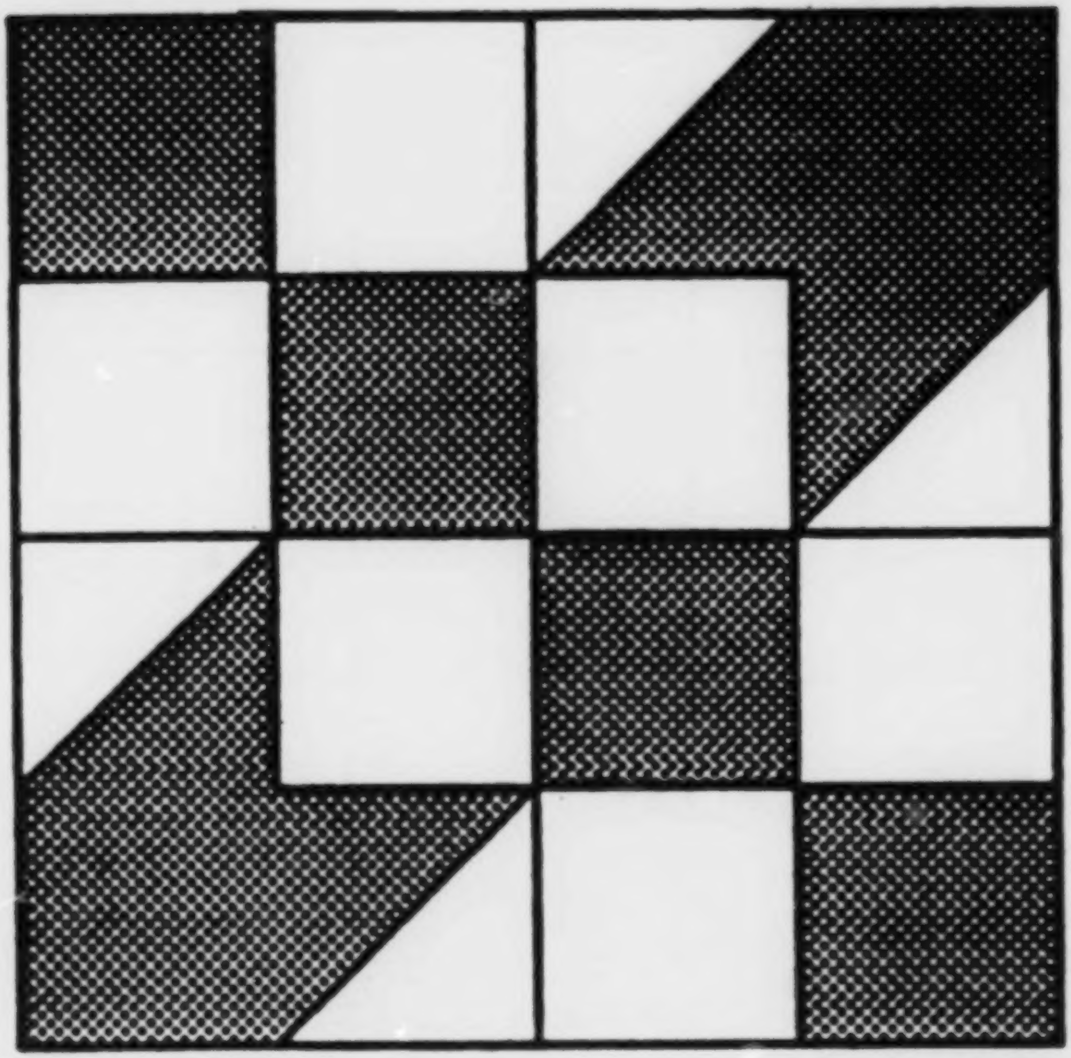
Activity Sheet 2.2 L

Churn Dash Quilt Pattern



Activity Sheet 2.5A

Road to Oklahoma Quilt Pattern

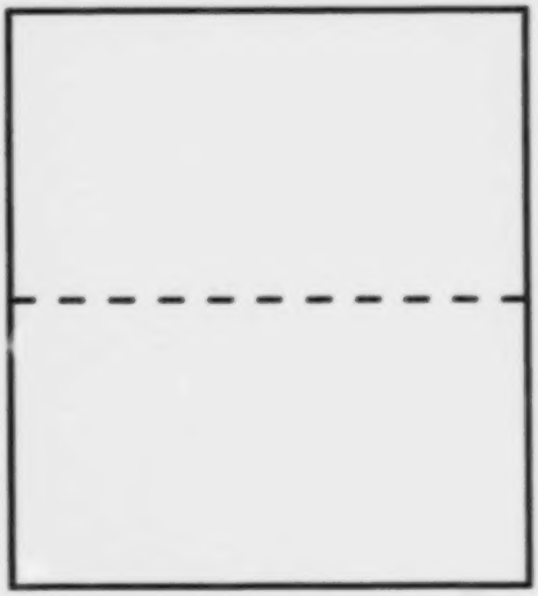


Activity Sheet 2.5A

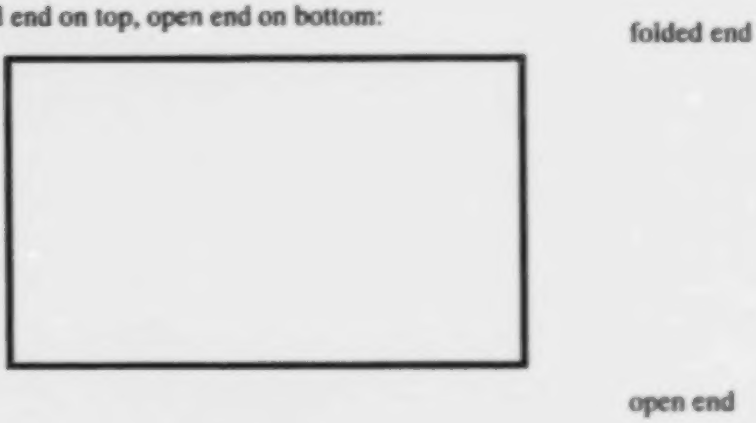
Peep Hole

Using a sheet of notebook paper, follow the diagrams below.

1) Fold the notebook paper on the dotted line:



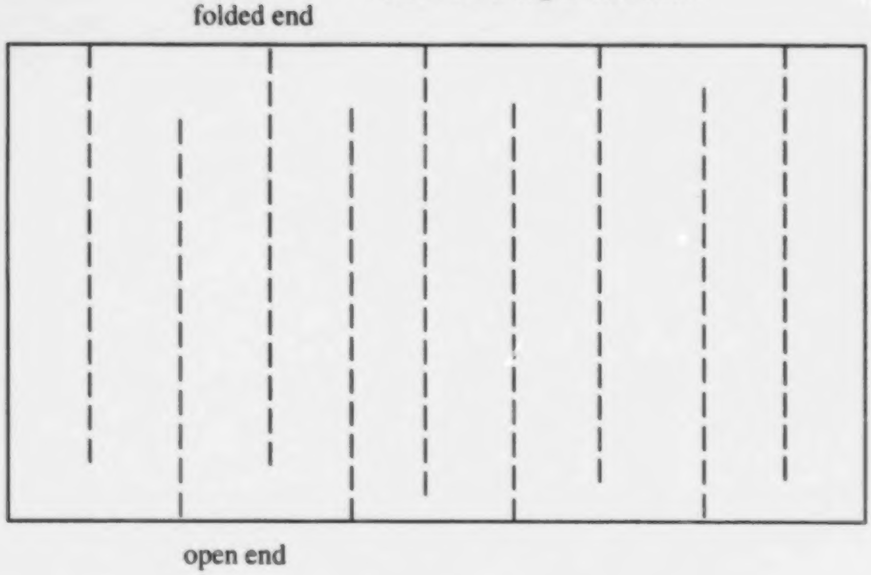
2) Place folded end on top, open end on bottom:



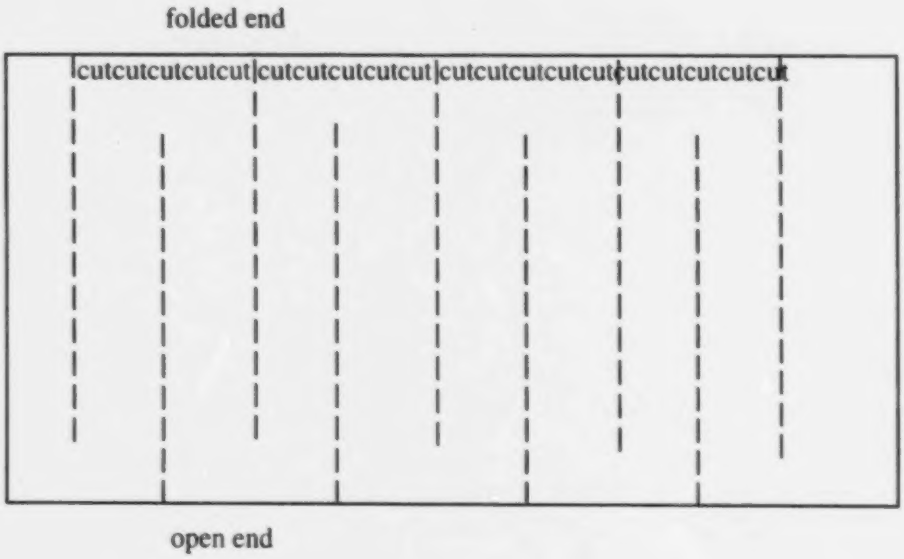
Activity Sheet 2.6 G

Solution:

Make slits along dotted lines:


folded end
open end

Then cut folded end as follows:


folded end
cutcutcutcutcut
cutcutcutcutcut
cutcutcutcutcut
cutcutcutcutcut
open end

48

SEVENTH GRADE GEOMETRY STRATEGIES

COMPETENCY GOAL 2: The learner will demonstrate an understanding and use of geometry.

2.1 Make constructions of perpendicular and parallel lines using a straightedge and compass.

A. MAC'S GROCERY STORE: Use listening activity to create "Mac's Grocery Store." Read the activity titled "Mac's Grocery Store" to the class. With ruler and compasses, the students will construct the design of Mac's store. **See Activity Sheet 2.1A**

B. COMMUNITY MAP: Using your own community or making up a new community, create a grid of streets using parallel and perpendicular lines. Name your streets.

C. TAG BOARD T-SHIRT: Design a T-shirt on tag board. Have students construct perpendicular and parallel lines to form a design on a tag board T-shirt. After constructing the design, students may use a variety of media to decorate their T-shirts.

D. MATH WALK: Take a five minute silent walk around an area of the school to look for examples of perpendicular and/or parallel lines. In your journal draw or construct your favorite example and write a brief description.

E. ALBUM COVER: Using parallel and perpendicular lines, students will design an album cover for their favorite musical group.

F. SOFTWARE: Have students use a geometric microcomputer program which enables them to do constructions of perpendicular and parallel lines. Some examples of geometric microcomputer programs are : Sunburst's Geometric preSupposer and Logo Writer.

G. DESIGN CONSTRUCTION: Using a compass and ruler, students reproduce given geometric designs. These designs should include more than one geometric shape. As students become more comfortable with reproducing these designs, they should produce designs of their own.

H. FABRIC SWATCH: Students use parallel and perpendicular lines to design a fabric swatch.

I. GOALPOST CONSTRUCTION: Give students a sheet of paper with one post drawn and have students construct the horizontal bar and then the second upright. Constructions should be done with a compass and straightedge. You may adapt this activity to a power line pole.

J. RECTANGULAR PRISM CONSTRUCTION: Using compass and straight edge, students construct three sets of congruent rectangles that will fit together to form a rectangular prism. Make these constructions on pieces of foam core board (available at frame shops). Cut out the rectangles. Use straight pins to put the pieces together to form a rectangular prism.

2.2 Use the concepts and relationships of geometry to solve problems.

A. RESTAURANT PUZZLE: You are working at a restaurant. In your section you have the following tables: 4 two-seat tables, 4 three-seat tables, and 3 five-seat tables. Four different groups come in for lunch with nine, ten, eleven and five people in the four groups. Arrange the tables to seat all four groups.

B. I HAVE...WHO HAS?: Use the sheet entitled "I Have…Who Has?" Write each line on an index card. The cards are distributed to the teacher and each student. The teacher reads the question, and then the student with the appropriate card reads his/her answer. That student then reads the question on his/her card and the game continues. **See Activity Sheet 2.2B**

C. WHO? WHAT ? HOW?: Use the sheet entitled, "Who? What? How?" and write each line on an index card. The cards are distributed to each student. Start the game by reading the statement and question at the top of the sheet. The student with the correct response will read his/her answer followed by the question on the card. Play continues until all cards are used. **See Activity Sheet 2.2C**

D. GEOMETRY WITH GUMDROPS: Supply each student with gumdrops and toothpicks. Demonstrate how a toothpick can represent a line going both directions. If a toothpick and a gumdrop can represent a ray, have students model other geometric figures such as right angles, obtuse angles, acute angles, cubes, prisms, pyramids, rhombuses.

E. CLOCK PUZZLE: Using MAPS A-45, students use a straightedge to determine how many rectangles can be drawn on a clock face. **See Activity Sheet 2.2E**

F. FIND THE "4's": Using MAPS B-44, students determine how many "4's" they can find in the figure. **See Activity Sheet 2.2F**

G. SQUARE IT!: Using MAPS B-45, students construct a square by identifying relationship of puzzle pieces. **See Activity Sheet 2.2G**

H. MYSTERY RECTANGLE: Using MAPS C-43, have students construct a rectangle whose area is eight square units. They are to use any of the given triangles to help them construct the rectangle. **See Activity Sheet 2.2H**

I. TRIANGLE TEASER: Using MAPS C-44 students will determine the number of triangles in the figure. **See Activity Sheet 2.2I**

J. CUBE NET: After giving each student a 3 x 4 rectangle, as in MAPS C-45, have the student remove unnecessary squares and then use the remaining squares to form a cube when the paper is folded on the given lines. **See Activity Sheet 2.2J**

K. CUBE WISE: Using MAPS C-46, students study the three different views of the same cube. They are to determine what figure is opposite the "X" side. **See Activity Sheet 2.2K**

L. TANGRAM PUZZLES: Using critical thinking skills, students recreate the tangram puzzles with tangram pieces. Puzzles are found in the appendix. **See Activity Sheet 2.2L**

M. FOLD-A-NET: The teacher is to prepare the activity by cutting apart the "Fold-A-Net" cards. Each column of four is a complete set. Put each set in an envelope. Two different sets are included. In one set the student will match the given net with the correct cube. In the second set the student will match the given cube with the correct net. **See Activity Sheet 2.2M**

N. GEOMETRY THINKER: Carefully read the clues that describe a polygon. Name that polygon in as few clues as possible. **See Activity Sheet 2.2N**

O. SOFTWARE/CONSTRUCTION: Using the Apple Logo program on your computer (or other drawing program) construct a triangle, hexagon, perpendicular lines, parallel lines or other geometric shapes.

P. PROBLEM CARDS: Use selected problems from "Problem Solving 1993, Set C" compiled by members of the TEAM (Teaching Excellence And Mathematics). These make good starters and ice breakers. **See Activity Sheet 2.2P**

Q. DONALD IN MATH MAGICLAND: You may wish to show "Donald in Math Magicland" by Disney. (This is often on the Disney channel.) This video covers geometry in nature, the golden ratio, Pool Table math, etc.

R. POOL TABLE MATH: MECC has a program called "Problem Solving Strategies" which includes a version of Pool Table math. This could be used in the classroom.

2.3 Use models to develop the concept of the Pythagorean Theorem.

A. GEOBOARD & PYTHAGORAS: Have students use geoboards to illustrate the Pythagorean Theorem. Place four geoboards together to make a large square. Use rubber bands to form a right triangle; then square each leg of the triangle with other rubber bands. Estimate the areas. Look for a pattern. **See Activity Sheet 2.3A**

B. CUT AND TAPE PROOF OF THE PYTHAGOREAN THEOREM:
Step 1: Color the squares different colors.
Step 2: Cut out pieces 1, 2, 3, 4, and 5.
Step 3: Fit these five pieces on c^2 to illustrate
$c^2 = a^2+b^2$. **See Activity Sheet 2.3B**

C. PYTHAGOREAN THEOREM WITH TILES: Using square tiles or graph paper the student will explore the Pythagorean relationship by working with a 3-4-5 right triangle. They can repeat the procedure with other Pythagorean triples. It would be helpful for them to keep a chart of their results with the following column headings: # of squares on side A; # of squares on side B; # of squares on side C. Ask students to look for a pattern ($a^2 + b^2 = c^2$). **See Activity Sheet 2.3C**

D. PAPER FOLDING AND MAKING TANGRAM PIECES:
Following the directions on the enclosed blackline masters, the teacher will lead the class in making the seven tangram pieces from a plain sheet of paper. **See Activity Sheet 2.3D**

E. TANGRAM TRIANGLE: Have students use tangram pieces to form a triangle with a square on the legs and hypotenuse of the right triangle. They will take the pieces of the two smaller squares to arrange them on the large square connected to the hypotenuse.

F. AMAZING PYTHAGORAS: Students can do the "Amazing Pythagoras" activity to develop the concept of the Pythagorean Theorem. Have students classify triangles, measure the sides of triangles (to the nearest half-centimeter), and square the sides. After recording the results in a table, students make observations and predictions. **See Activity Sheet 2.3F**

G. ARE THEY RIGHT?: Draw nine triangles (of which six are right triangles) of different sizes. Label the triangles A through I. Have students measure the sides of each triangle and record the data in a table. Students will then square the lengths of the sides and record this data in the table. After studying the results, they are to write about any observations they have made.

2.4 Identify applications of geometry in the environment

A. QUILT PATTERNS: Students can use wallpaper scraps (discarded wallpaper books) and quilting designs to make their own 18" x 24" quilt square.

B. GEOMETRY SCAVENGER HUNT: Go on a scavenger hunt of the school grounds to photograph or video examples of geometry. A student-produced video can be integrated with other areas of the curriculum.

C. GEOMETRY HUNT: Take students outside for an around-the-school tour. In their math journals students jot down the different geometric figures that they see on the tour. Return to the classroom and discuss your findings.

D. NEWSPAPER ACTIVITY: Have students cut a picture from a newspaper or a magazine. Using a marker, they are to trace examples of parallel and perpendicular lines. After class discussion, display work.

E. LOGO DESIGN: Have students design a logo for a geometry conference, a local business, or a school team.

F. GEOMETRY IN ARCHITECTURE: Students are to go outside and observe the architecture/design of the front of the school. Consider what an architect would need to know about geometry in designing a building. Give examples of geometric principles used in architecture.

G. ENLARGING A DRAWING: Often, in advertising, logos need to be enlarged. Have students enlarge a drawing using the following procedure. Take a simple drawing and enlarge it several times using a point of projection. Choose a point of projection (P). Draw lines from Point P through each vertex on the polygon (or identified points on the logo) extending out past the vertex. Measure the distance from P to vertex A. Mark off the same distance on the line. Call it A' (A prime). Repeat for each vertex. Connect the vertices. (Distances can be doubled, tripled etc. for greater enlargements). **See Activity Sheet 2.4G**

H. GEOMETRY COLLAGE: Students are to collect pictures from magazines and other sources to design a collage of geometry in the environment: food, clothing, sports, advertising entertainment, space, etc.

I. MIRROR GEOMETRY: Using a wall-type mirror beside a doorway, students do creative movement as they stand in the doorway and show symmetry in motion using one leg, one arm, one hand, etc. The motion would show symmetry as well as reflection. These movements could be videotaped and played back for the class.

J. GARDEN DESIGN: Using a compass and ruler, students design a circular, symmetrical flower garden.

K. VIDEO SERIES: "Futures II" is a series of videos that show how important mathematics is to the students' futures. Famous personalities are in each 15-minute fast-moving video which is narrated by Jaime Escalante. These are available from PBS and may be taped by your media person.

2.5 Given models of 3-dimensional figures, draw representations.

A. QUICK DRAW ACTIVITIES: Make transparencies for the quick draw materials from MAPS Plus. Show three-dimensional shape for three seconds and ask students to draw what they saw. Give students another three second viewing of the shape to complete and correct their drawing. Continue as necessary until all students have completed the drawing. Some will have to see it several times. Finally, allow students to discuss how they remembered the picture to do their drawing. **See Activity Sheet 2.1A**

B. 3-D MURAL: Have students check out instructional drawing books. Class will participate in 3-dimensional drawing practice on a large sheet of paper to prepare mural.

C. DRAW A BOX: Have students bring in small boxes from home. (Ex. tissue, aspirin, or toothpaste boxes) These models will be used to draw orthographic and isometric representations. Models may also be drawn from prisms built with cubes.

D. 19 PROBLEM EXTENSION: Extend the "19 Problem" from Creative Publications found in Linking Curriculum, Instruction, and Assessment: The Geometry Strand (pg. 56-57) to a problem of packaging and shipping in the most economical way. Assign the following problem to the students. A store is ordering cases of blankets that are 2m x 2m x 4m, cases of sheets that are 2m x 2m x 2m and cases of pillow cases that are 1m x 1m x 1m. A truck has a trailer 4m x 4m x 8m. If the store wants to order approximately the same number of cases of each item and completely fill the truck, how many of each should the store order? Justify your answer. Make either an orthographic drawing or an isometric drawing of your solution. (This also addresses objectives 2.2 and 2.4.)

E. BIRD'S EYE VIEW: The following activity can be used as a student assignment. Do a drawing of your bedroom, home, classroom, or school to represent a bird's eye (orthographic) view of your environment.

F. DESIGN AND DRAW: Give students interlocking cubes. Have them design a building with the cubes. Have students draw a front view, side views, and a back view of their building. Give the students isometric dot paper and have them draw a corner view of their building. (This is a bit more difficult to do.) The isometric paper is for the corner view.

G. 3-D ORIGAMI: Show students 3-D origami models and have them draw representations of these models.

H. 3-D DRAWING: Give students a drawing or model of a 3-D figure. Have them draw the end, side, and top views of the figure. **See Activity Sheet 2.5H**

I. GOING UP: Students use blocks to build 3-dimensional models from clues shared in a collaborative group setting. After completing the task, individual students draw an orthographic or isometric representation of the model. **See Activity Sheet 2.5I**

J. OPPOSITES ATTRACT: Show students three views of the same cube and have them identify opposite faces. **See Activity Sheet 2.5J**

2.6 Given the end, side, and top views of 3-dimensional figures, build models.

A. GEOBLOCK ID: Using geoblocks, teacher will trace each different face on a task card and identify the number of faces for each individual geoblock. Have the students match the geoblock to the task card.

B. BUILD A GEOBLOCK MODEL: Using two or three geoblocks or cubes, the teacher will build models of three-dimensional figures and draw the end, side and top views of these models on task cards. Have students build these models from the drawings.

C. BUILDING A MODEL: Students are to cut apart the six large squares and use them to build a model of the 3-dimensional figure that the given net will make. **See Activity Sheet 2.6C**

D. CONSTRUCT 3-D ORIGAMI MODEL: Have students construct 3-D origami models using formats found in origami books like Easy Origami by John Montroll and Easy and Fun Paper Folding by Johanna Huber and Christel Claudius.

E. TRIANGULAR PYRAMID: Students will build a model of the 3-D triangular pyramid shown by cutting out the four triangles given to make the model. **See Activity Sheet 2.6E**

F. BUILD A MODEL: Given the side, top and end views of a 3-D figure, students are to build a model of the figure with cubes. **See Activity Sheet 2.6F**

G. AERIAL VIEW PHOTOGRAPHY: Students are to find aerial view photographs of buildings (in or around your area or school if possible) and discuss how these buildings may look from the ground. They may then build models of buildings.

H. SOMA CUBES: Soma Cubes is a puzzle invented by a Danish writer, Piet Hein. Have students make puzzles pieces out of interlocking cubes. No two pieces are alike. Students may use the interlocking cubes to build 12 different models. **See Activity Sheet 2.6H**

2.7 Graph on a coordinate plane shapes and congruent figures.

A. WORKING BACKWARDS: Have students draw a simple picture on graph paper then write the coordinates of the points on another piece of paper. Exchange the list of the coordinates of the points with another student and each student is to draw a picture to correspond to the given points. Check by comparing the pictures to the originals.

B. PICTURE PLOT: Using a bulletin board, make a large coordinate graph (laminate). On separate pieces of paper list the coordinates of the points of a desired picture (one coordinate to a piece of paper); however, omit some of the coordinates. Have students select a point to be plotted and put it on the graph. Geometric shapes may be used as "points." Then have students decide which points are missing and complete the picture.

C. CONGRUENCY MATCH: Gives students a graph of two figures and ask them to decide of they are congruent. Is figure ABCD congruent to figure EFGH? Ask students to give coordinates of another figure that is congruent to figure ABCD. An additional set of figures in included. **See Activity Sheet 2.7C**

D. TRANSLATIONS & CONGRUENCY: Slide each figure five units left and five units along the y-axis. Graph a congruent figure at these new coordinates. **See Activity Sheet 2.7D**

E. TRACING REFLECTIONS: Give students a shape on a coordinate plane, and have them trace it and the axes on wax paper so it may be reflected over the x- or y- axis. Flip the wax paper across the axis. Plot the new vertices and complete the figure. **See Activity Sheet 2.7E**

F. "MIRA" ACTIVITY: Using a mirror or "mira," students will reflect figures across the x- or y- axis, and draw the reflection (a congruent figure). They are to list the coordinates for the original figure and the reflected figure. **See Activity Sheet 2.7F**

G. CONSTRUCTING GEOMETRIC FIGURES: Using graph paper students will construct figures according to given specifications. They will then record the coordinates of the verticies of the figures. **See Activity Sheet 2.7G**

References and Additional Resources

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5. Geddes, Dorothy, Geometry In the Middle Grades: Curriculum and Evaluation Standards for School Mathematics, Addenda Series, Grades 5-8, National Council of Teachers of Mathematics, Inc., Reston, Virginia, 1992.

6. Gillespie, N.J., Mira Activities for Junior High School Geometry, Creative Publications, Sunnyvale, California, 1973.

7. Holden, Linda, Thinker Tasks - Critical Thinking Activities, Creative Publications, Sunnyvale, California.

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BLACKLINE MASTERS
SEVENTH GRADE
STRATEGIES

Mac's Grocery Store
Activity Sheet 2.1A

Mac is building a grocery store. Listen to my directions as I tell you what to construct to represent a design for the store.

Construct a rectangle 15 centimeters wide and 20 centimeters long. Turn the paper so that the 15 centimeters lines are on the top and bottom of your paper. This is Mac's store. Draw a representation of a front door on the short line at the top of your paper. This is the left corner area.

Beginning 3 centimeters from the top, construct a line with a straight edge 14 centimeters long and parallel to the side wall of the store. This is aisle 1; it is 3 centimeters from the left wall. Now construct a congruent parallel line 3 centimeters to the right of aisle 1. This is aisle 2. Construct aisles 3 and 4 in the same way. Aisle 5 should be constructed perpendicular to the other aisles and at the bottom of the drawing.

The counter is at the front of the store. The counter is 2 centimeters wide and 10 centimeters long and is at the top right hand corner of the store.

I Have…Who Has?

Activity Sheet 2.2B

I HAVE	**WHO HAS**
GEOMETRY	an angle whose measure is greater than 0° and less than 90°
ACUTE ANGLE	the number of angles in a rectangle
4	half a sphere
HEMISPHERE	the measure of any angle of an equilateral triangle
60°	the point that two or more rays, sides or edges have in common
VERTEX	the number of degrees in a circle
360	a four-sided polygon with exactly one pair of parallel lines
TRAPEZOID	the measure of a right angle
90°	a four-sided polygon that has four right angles and four sides that are the same length
SQUARE	the number of sides a decagon has
10	part of a straight line with an end point
RAY	a triangle that has one right angle
RIGHT TRIANGLE	a tool used to measure angles
PROTRACTOR	two lines that intersect at right angles to each other
PERPENDICULAR LINES	the sum of the lengths of the sides of a polygon
PERIMETER	two rays having a common end point

I Have...Who Has?

Activity Sheet 2.2B continued

I HAVE	WHO HAS
ANGLE	the distance around a circle
CIRCUMFERENCE	two points on a line and all the points between them
LINE SEGMENT	an angle whose degree measure is greater than 90° and less than 180°
OBTUSE ANGLE	an eight-sided polygon
OCTAGON	two lines which lie in the same plane and do not intersect
PARALLEL LINES	a flat surface
PLANE	a triangle with three lines of symmetry
EQUILATERAL TRIANGLE	a quadrilateral with its opposite sides parallel
PARALLELOGRAM	a triangle with one line of symmetry
ISOSCELES TRIANGLE	a five sided polygon
PENTAGON	two angles having the same measure
CONGRUENT ANGLES	the unit for measuring angles
DEGREES	a triangle with no lines of symmetry
SCALENE TRIANGLE	a parallelogram with four right angles and opposite sides congruent
RECTANGLE	the study of space and shapes

Developed by Barbara Barker
Hazelwood Elementary School

WHO? WHAT? HOW?

Activity Sheet 2.2C

I have the polygon that has the least number of sides. What is it called?

QUADRILATERAL - What is a ten-sided polygon called?

PERIMETER - What do you call the surface within a polygon?

RAY - How many of these make an angle?

RECTANGLE - Suppose a polygon has opposite sides congruent and parallel, but no 90° angles. Name it!

SEGMENT - What do you call half of a line?

RIGHT - How many angles in a triangle?

IMPOSSIBLE - What is a four-sided polygon called?

DECAGON - What do you call a four-sided polygon with all sides equal and 90° angles?

TRIANGLE - What is a five-sided polygon called?

DIAMETER - What do you call half of a diameter?

PARALLELOGRAM - Suppose a polygon is four-sided and all sides are equal, but it does not necessarily have right angles. Name it!

HEXAGON - What is a two-sided polygon called?

AREA - What do you call a plane figure that is perfectly round?

CHORD - What do you call part of a line with two or more points?

THREE - Describe what you have if you have a triangle.

CIRCUMFERENCE- What do you call part of the circumference?

TWO - What is a 90° angle called?

RADIUS - What do you call the distance around a circle?

SQUARE - What do you call a four-sided polygon with opposite sides equal and parallel and all right angles?

ARC - What do you call the line segment connecting the endpoints of an arc?

CIRCLE - What do you call the line segment that intersects the center of a circle?

PENTAGON - What is a six-sided polygon called?

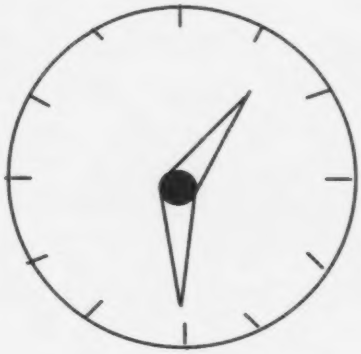
RHOMBUS - What do you call the distance around a polygon?

MAPS Problem Solving

Activity Sheet 2.2E

Problem A - 45

How many different rectangles can be drawn using the 12-hour marks on a clock face as corners?

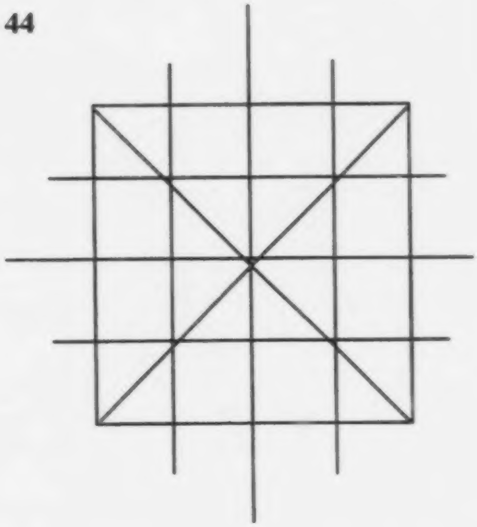


MAPS Problem Solving

Activity Sheet 2.2F

Problem B - 44

How many times can you find a "4" in the figure?



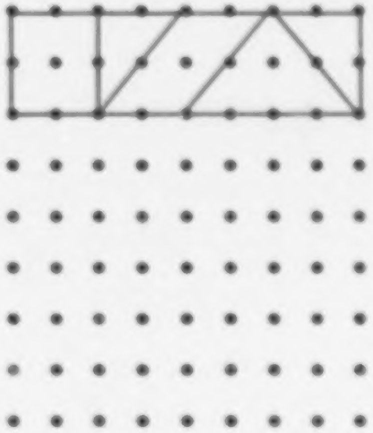
68

MAPS Problem Solving

Activity Sheet 2.2G

Problem B - 45

Rearrange the five pieces which form this rectangle so that they form a square. Record your results on the grid provided.

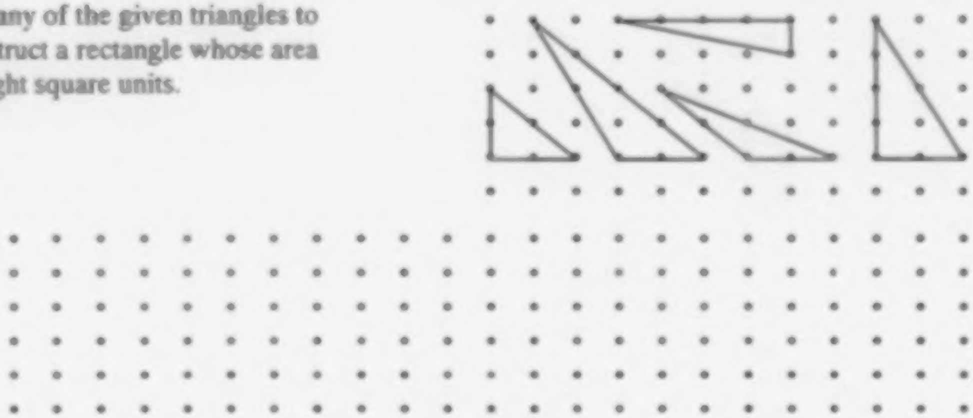


MAPS Problem Solving

Activity Sheet 2.2H

Problem C-43

Use any of the given triangles to construct a rectangle whose area is eight square units.



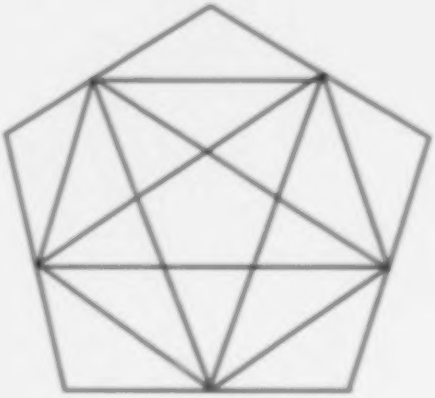
70

MAPS Problem Solving

Activity Sheet 2.21

Problem C - 44

How many triangles are in this figure?

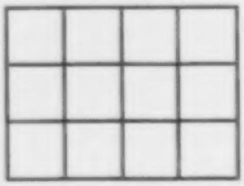


MAPS Problem Solving

Activity Sheet 2.2J

Problem C - 45

A cube is a solid figure which has six square faces. Show by marking with X's the six squares you would cut from the pattern so that the remaining six squares will form a cube when the paper is folded along the given lines.

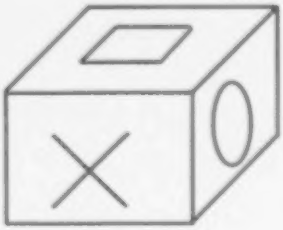


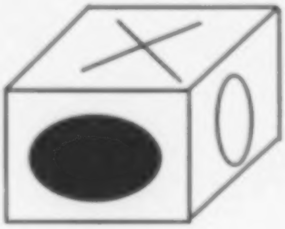
MAPS Problem Solving

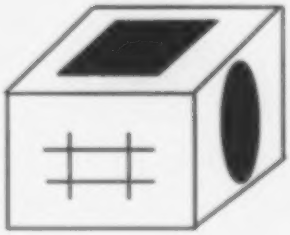
Activity Sheet 2.2K

Problem C - 46

Look at these three different views of the same cube. What figure is on the side of the cube which is opposite the side with the "X" on it?



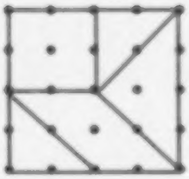




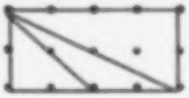
MAPS-ANSWERS
Activity Sheet 2.2 E-K

A-45	15 rectangles
B-44	44 times

B-45



C-43

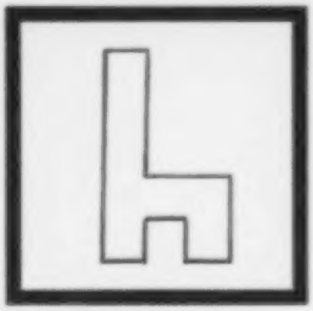


C-44 40 triangles

C-45

		X	X
X			X
X	X		

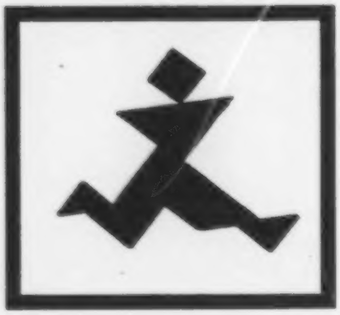
C-46 "#" is opposite the "X"



TANGRAM PUZZLE SHEET

Activity Sheet 2.2L

Use tangram pieces to create the picture in the left hand corner. Use the space below for your drawing. .

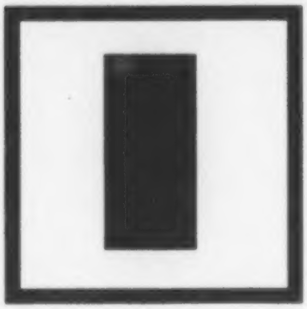


TANGRAM PUZZLE SHEET

Activity Sheet 2.2L

Use tangram pieces to create the picture in the left hand corner. Use the space below for your drawing.

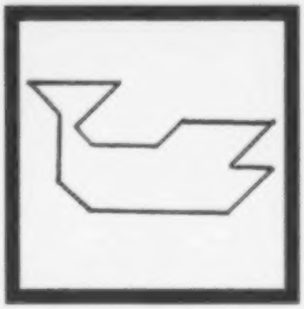
76



TANGRAM PUZZLE SHEET

Activity Sheet 2.2L

Use tangram pieces to create the picture in the left hand corner.
Use the space below for your drawing.



TANGRAM PUZZLE SHEET

Activity Sheet 2.2L

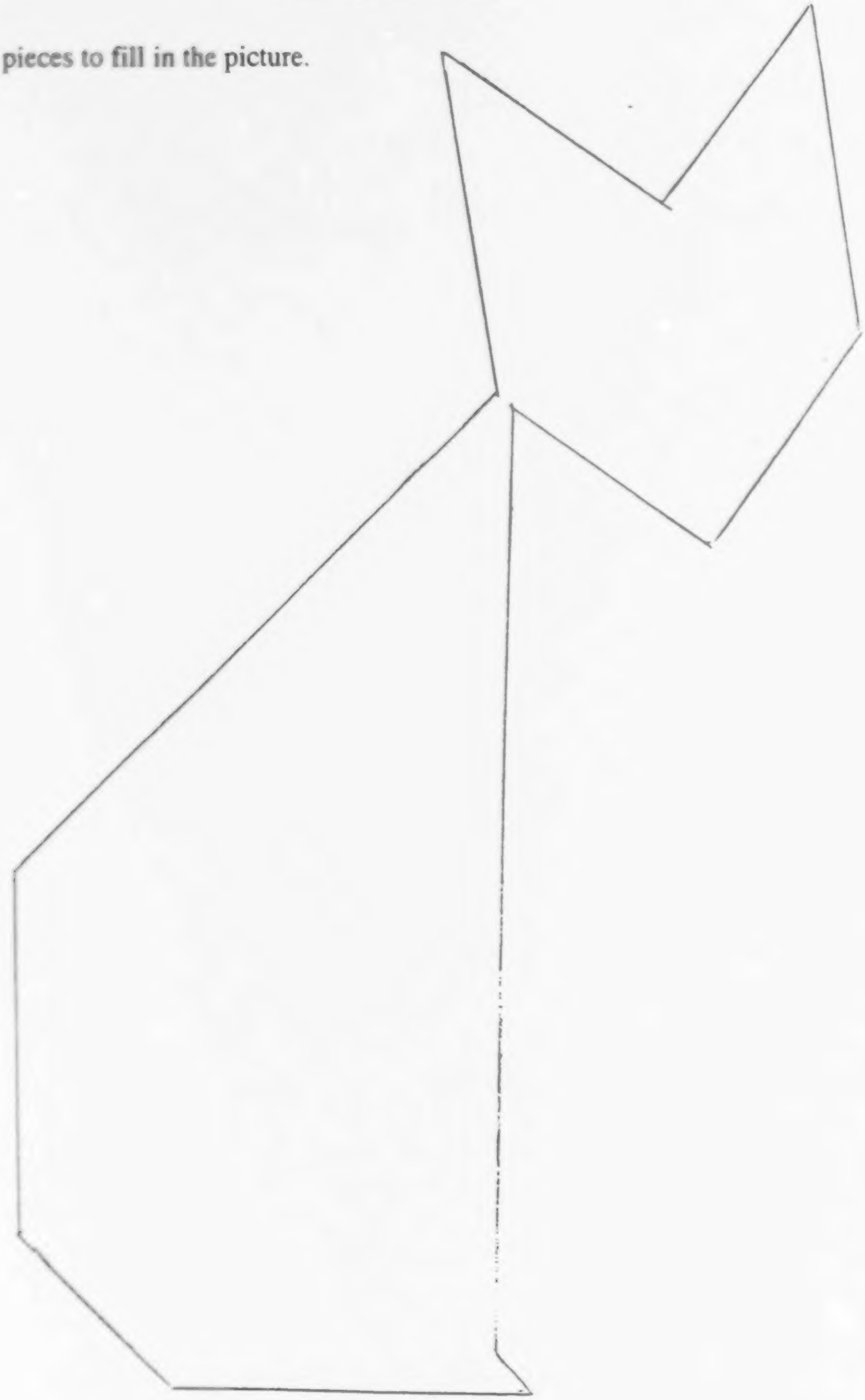
Use tangram pieces to create the picture in the left hand corner. Use the space below for your drawing.

78

TANGRAM PUZZLE SHEET

Activity Sheet 2.2L

Use the tangram pieces to fill in the picture.

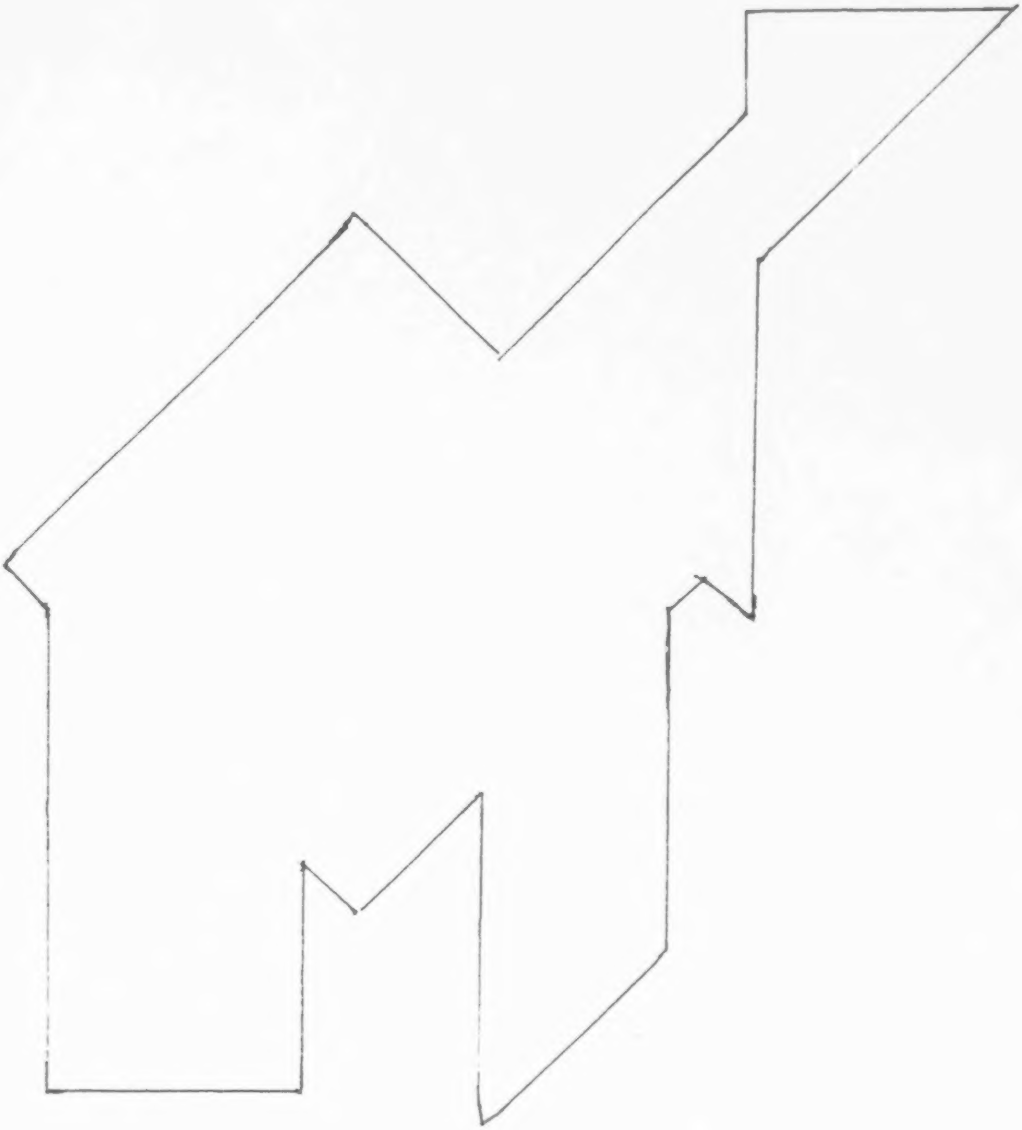


Alley Cat

TANGRAM PUZZLE SHEET

Activity Sheet 2.2L

Use the tangram pieces to fill in the picture.

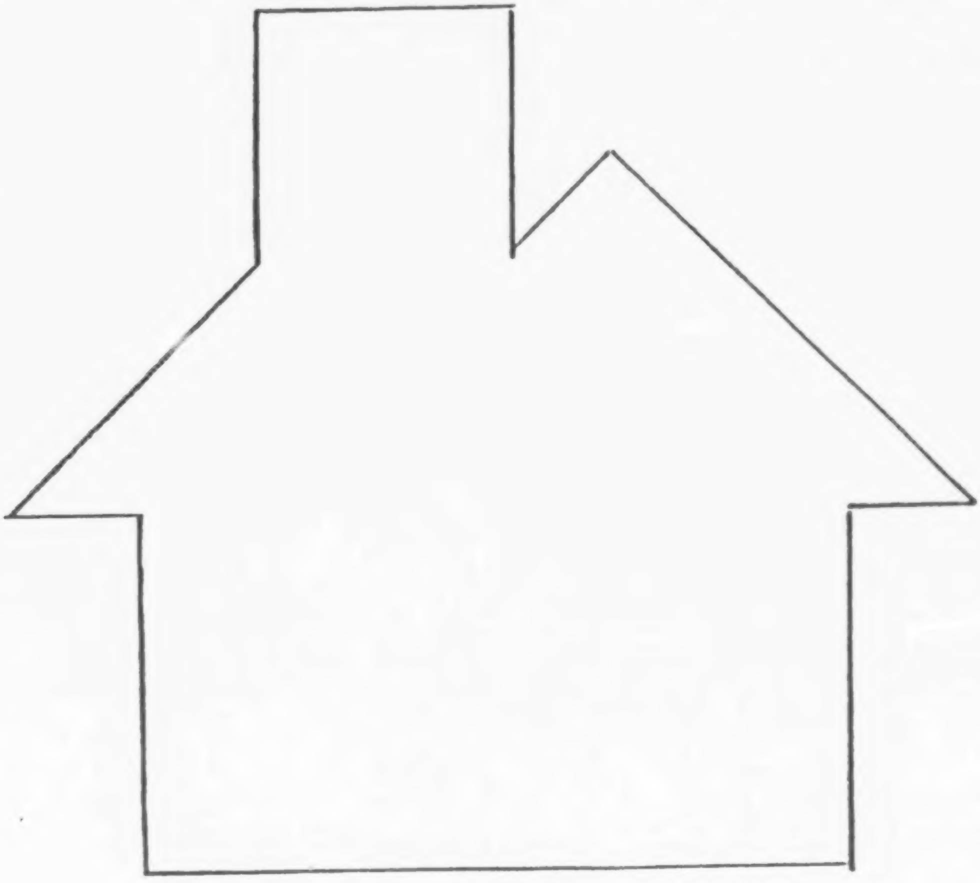


Dynamic Camel

TANGRAM PUZZLE SHEET

Activity Sheet 2.2L

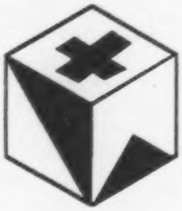
Use the tangram pieces to fill in the picture.

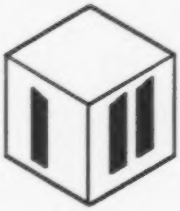


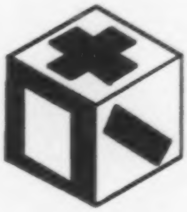
THE HOUSE

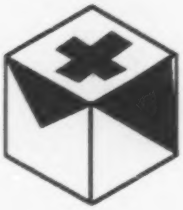
Activity Sheet 2.2M
Fold A Net Cards

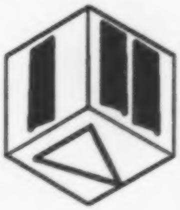
Choose the cube made by the net. -- "Fold a Net" Cards

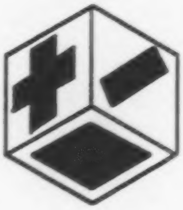


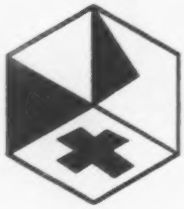


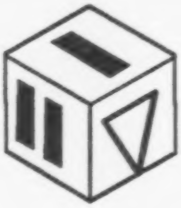


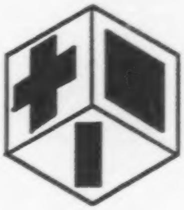


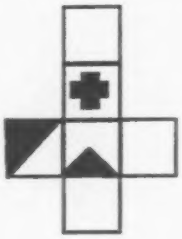


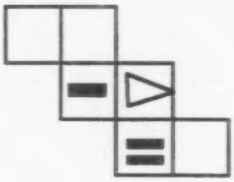


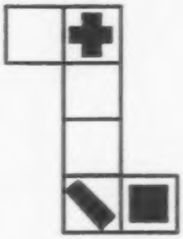






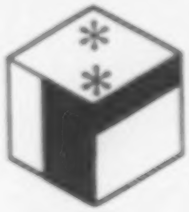


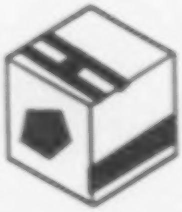


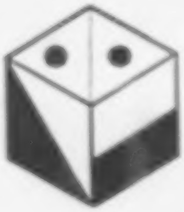


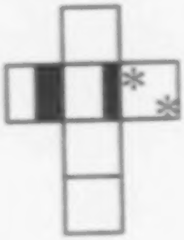
Activity Sheet 2.2M
Fold A Net Cards

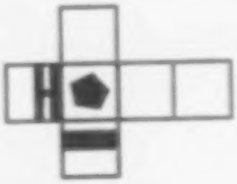
Choose the net that makes the cube -- "Fold a Net" Cards

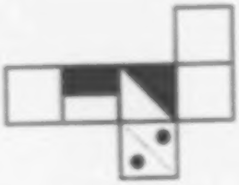


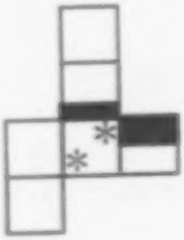


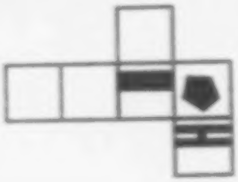


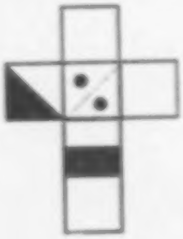


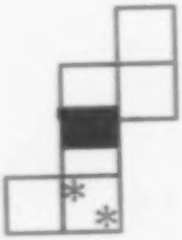


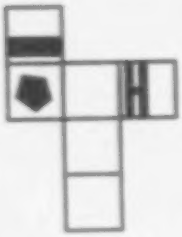


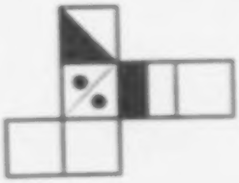












Geometry "Thinker"

Activity Sheet 2.2N

A. It is a polygon with four straight sides.

B. Two of its sides are parallel.

C. Its other two sides are parallel.

D. It has two angles that are congruent.

E. It has two more angles that are congruent.

F. Its diagonals bisect each other.

G. It has a 90° angle.

H. Its diagonals do not form 90° angles.

I. Its diagonals are congruent.

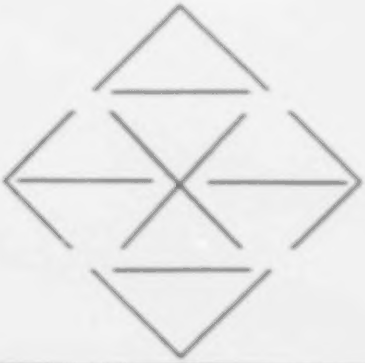
J. All of its angles are 90° angles.

Problem Solving
Activity Sheet 2.2P

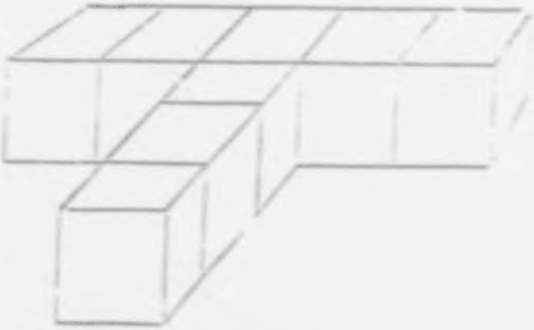
A 3" x 3" x 5" rectanglar prism is painted orange on all sides. Draw this figure. If the prism is cut into one inch cubes, how many cubes are there? How many of the cubes are painted orange on four faces? three faces? two faces? one face? no faces?

Assume that the sides of triangles are limited to whole number measures. How many different triangles have a perimeter of 12? Explain how you determined this.

There are 16 toothpicks in this design. Remove four toothpicks so that only four congruent triangles remain.



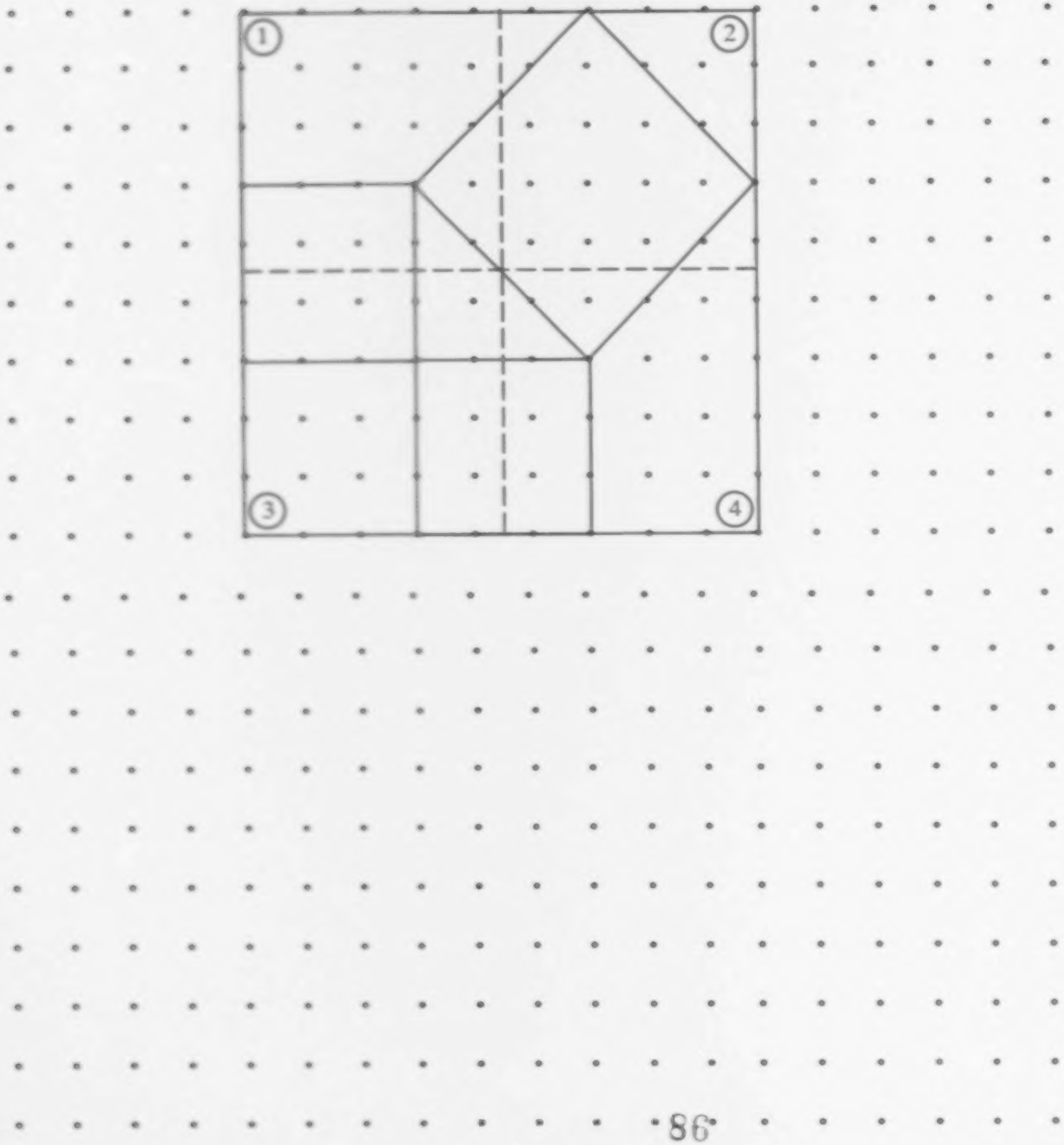
Eight 1-inch cubes are put together to make a "T." If the complete figure is painted blue, how many cubes have exactly four blue faces?



How are the remaining cubes painted?

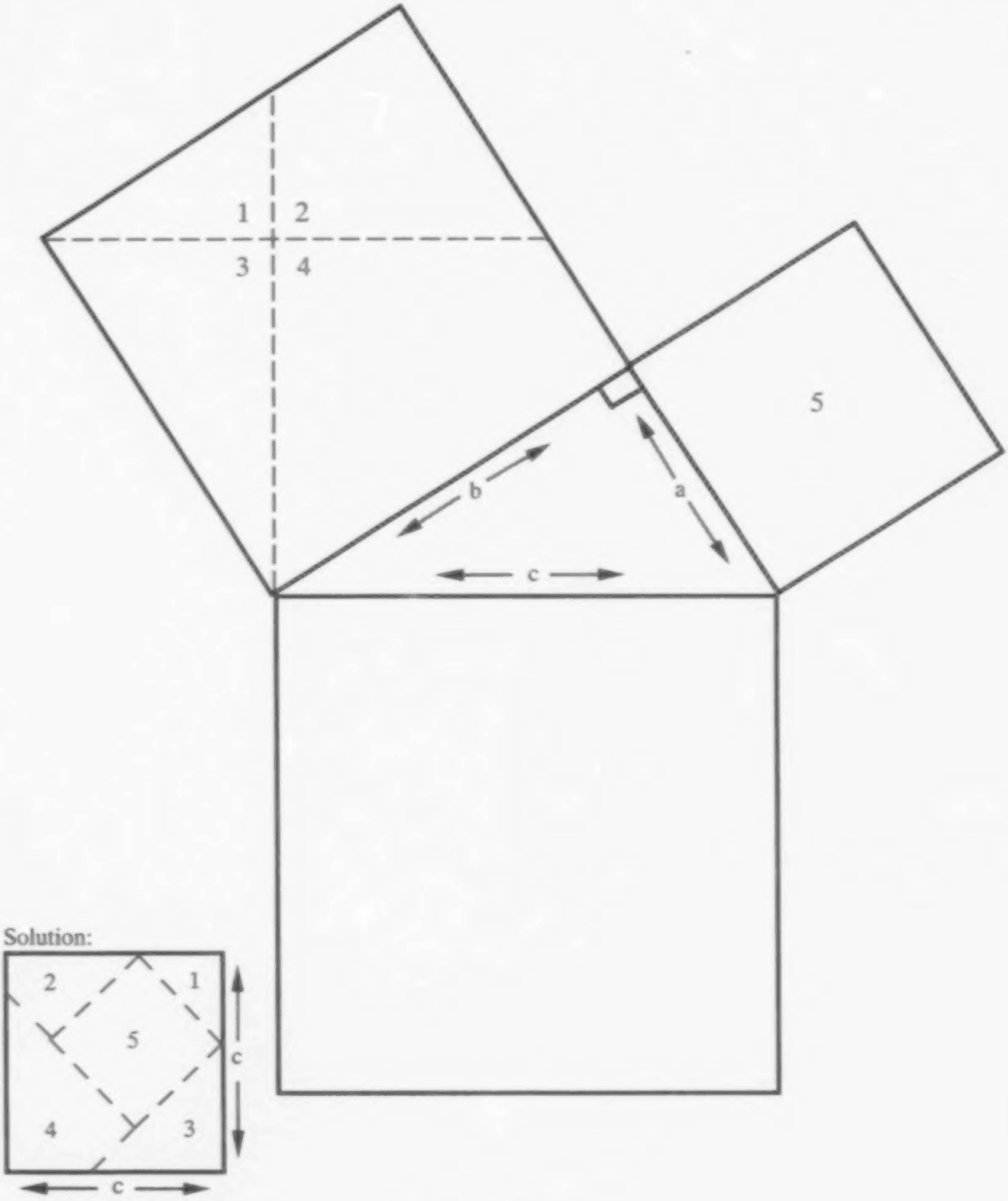
GEOBOARD & PYTHAGORAS

Activity Sheet 2.3A



CUT AND TAPE PROOF OF THE PYTHAGOREAN THEOREM

Activity Sheet 2.3B


1
2
3
4
5
b
a
c
2
1
5
4
3
c
c

Solution:

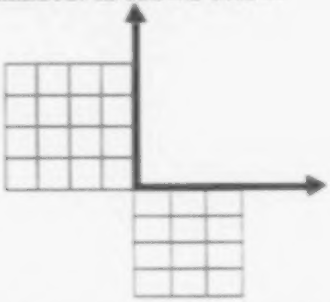
PYTHAGOREAN THEOREM WITH TILES

Activity Sheet 2.3 C

OBJECTIVE: The student will develop the Pythagorean Theorem using square tiles/graph paper.

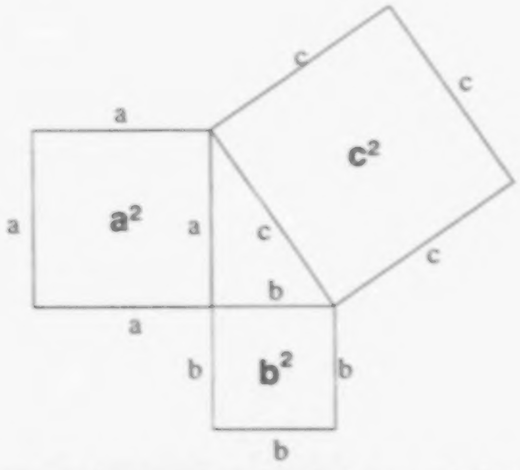
MATERIALS: Square tiles, right triangle handout

PROCEDURE: Have students work in groups of four. Provide each group with at least 50 square tiles and a handout containing a large right triangle. Instruct the students to construct a square using 9 tiles and another square using 16 tiles. Tell the students to place the two squares corner to corner along the legs of the right triangle on the handout as shown below.



Challenge students to find how many tiles it will take to construct a third square in order to complete the triangle.

$$\begin{aligned} a^2 + b^2 &= 4^2 + 3^2 \\ &= 16 + 9 \\ &= 25 \\ &= c^2 \\ 5 &= c \end{aligned}$$

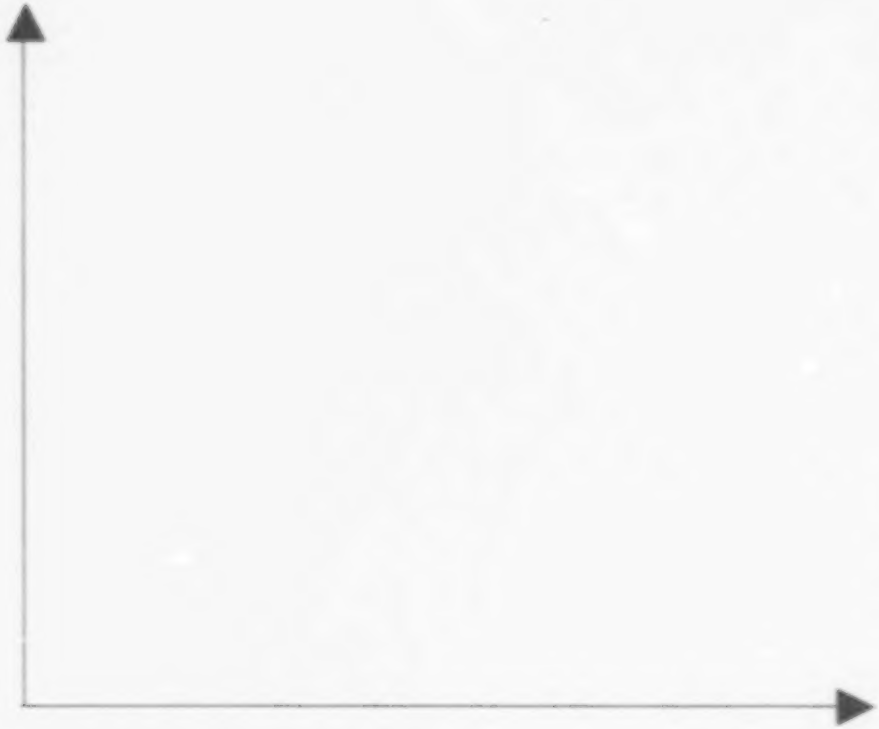


Have students repeat the steps for other Pythagorean Triples.

EVALUATION: Have students find the hypotenuse of right triangles when the two legs are known. Discuss and record results.

PYTHAGOREAN THEOREM WITH TILES

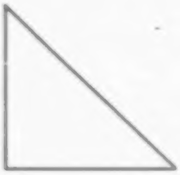
Activity Sheet 2.3 C



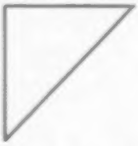
Paper Folding and Making Tangram Pieces

Activity Sheet 2.3D

Materials Needed: plain letter-sized paper
scissors **optional***

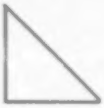


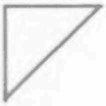
* Folding and tearing the pieces (rather than cutting) works well, especially if you "lick" the edge of the fold before tearing.



Follow the step-by-step directions on the following page to make the seven tangram pieces from a plain sheet of paper. At each step along the way, it is helpful if you fold and tear a large piece of paper as a **demonstration.**

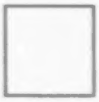
Discuss the pieces that are formed during each step and their relationship to each other. For example, after step B, the two smaller triangles are congruent to each other and together are equal in area to (or make-up) the larger triangle. Also, all three triangles are **right triangles.**

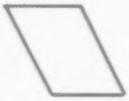




Introduce in the discussion the names of the different shapes as you proceed. This is a good opportunity to discuss the characteristics of **squares, triangles, parallelograms, trapezoids, and quadrilaterals.**

When finished making the **seven** tangram pieces, discuss the sizes of each piece relative to the others. For example, 2 small triangles can be put together to make the square or the parallelogram.





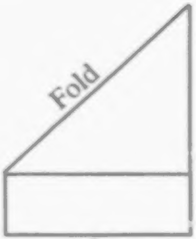
Activity: Ask the students to put the pieces back together to make a square. Make a rectangle. Make a triangle.

TANGRAM FOLD

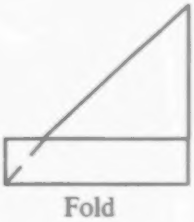
Activity Sheet 2.3D

First make a square from a rectangle. Give each student a rectangular piece of paper. Fold and cut as follows:

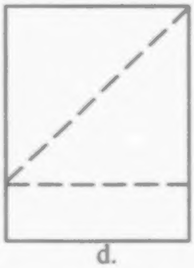
a.



b.



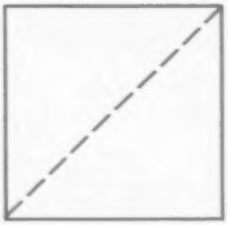
c.



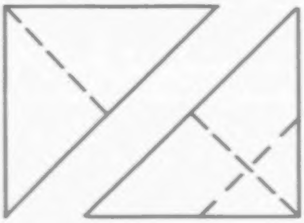
d.

Unfold
and cut

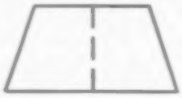
Now use the square to make a set of tangram pieces. Cut and fold as instructed below:



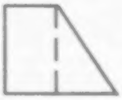
a.



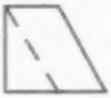
b. c.



d.



e.



f.

a. Fold square into two large congruent triangles. Cut apart on the fold.

b. Fold both large triangles into two right triangles. Cut one of them on the fold and set these two pieces aside.

c. Fold the second large triangle so that the right angle corner touches the midpoint of the base. Unfold and cut the triangle from the trapezoid. Set the triangle aside.

d. Cut the trapezoid in half (on the fold).

e. Fold one of the quadrilaterals into a square and a right triangle. Cut apart and set these pieces aside.

f. Fold the remaining quadrilateral into a parallelogram and a right triangle. Cut along the fold to complete the set.

THE AMAZING PYTHAGORAS

Activity Sheet 2.3F

Mathematics teaching objectives:

- Identify right triangles.
- Discover the Pythagorean Property for right triangles.

Problem-solving skills pupils might use:

- Make necessary measurements for obtaining a solution.
- Use a table.
- Look for patterns.
- Make generalizations based upon data.

Materials needed:

- Centimeter ruler

Comments and suggestions:

- This lesson should be used as a class activity. Many students will need some teacher direction.
- All measurements in the triangles, except E and I, are whole numbers when measured to the nearest half-centimeter.
- In problem 3, some pupils may need a hint for finding a pattern in the table. Suggest they add the numbers in the a^2 and b^2 column for each triangle.
- An effective overhead visual can be used for presenting an informal proof of the Pythagorean relation.

Answers:

1. & 2.

Triangle	a	b	c	a^2	b^2	c^2
√A	5	12	13	25	144	169
√B	3	4	5	9	16	25
C	4	4	5	16	16	25
D	2	2	2	4	4	4
√E	2	1.5	2.5	4	2.25	6.25
√F	6	8	10	36	64	100
√G	8	15	17	64	225	289
H	3	6	8	9	36	64
√I	7.5	10	12.5	56.25	100	156.25

3. $a^2 + b^2 = c^2$

4. No, because $81 + 100 \neq 121$.

5. Yes, because $81 + 144 = 225$.

THE AMAZING PYTHAGORAS

Activity Sheet 2.3F

Needed: Centimeter ruler

Pythagoras was a Greek mathematician who lived about 2500 years ago. He made many mathematical discoveries. One of his most famous discoveries was about certain kinds of triangles.

Study the triangles on the next page. Triangles A, B, E, F, G, and I are particular kinds of triangles. Draw another triangle that also fits this category.

Place a check in front of the 6 letters in the table that represent right triangles.

1. Measure the sides of each triangle to the nearest half-centimeter. Place your measurements in the left part of the table.

Triangle	a	b	c	a^2	b^2	c^2
A						
B						
C						
D						
E						
F						
G						
H						
I						

2. Make the computations necessary to complete the right side of the table.

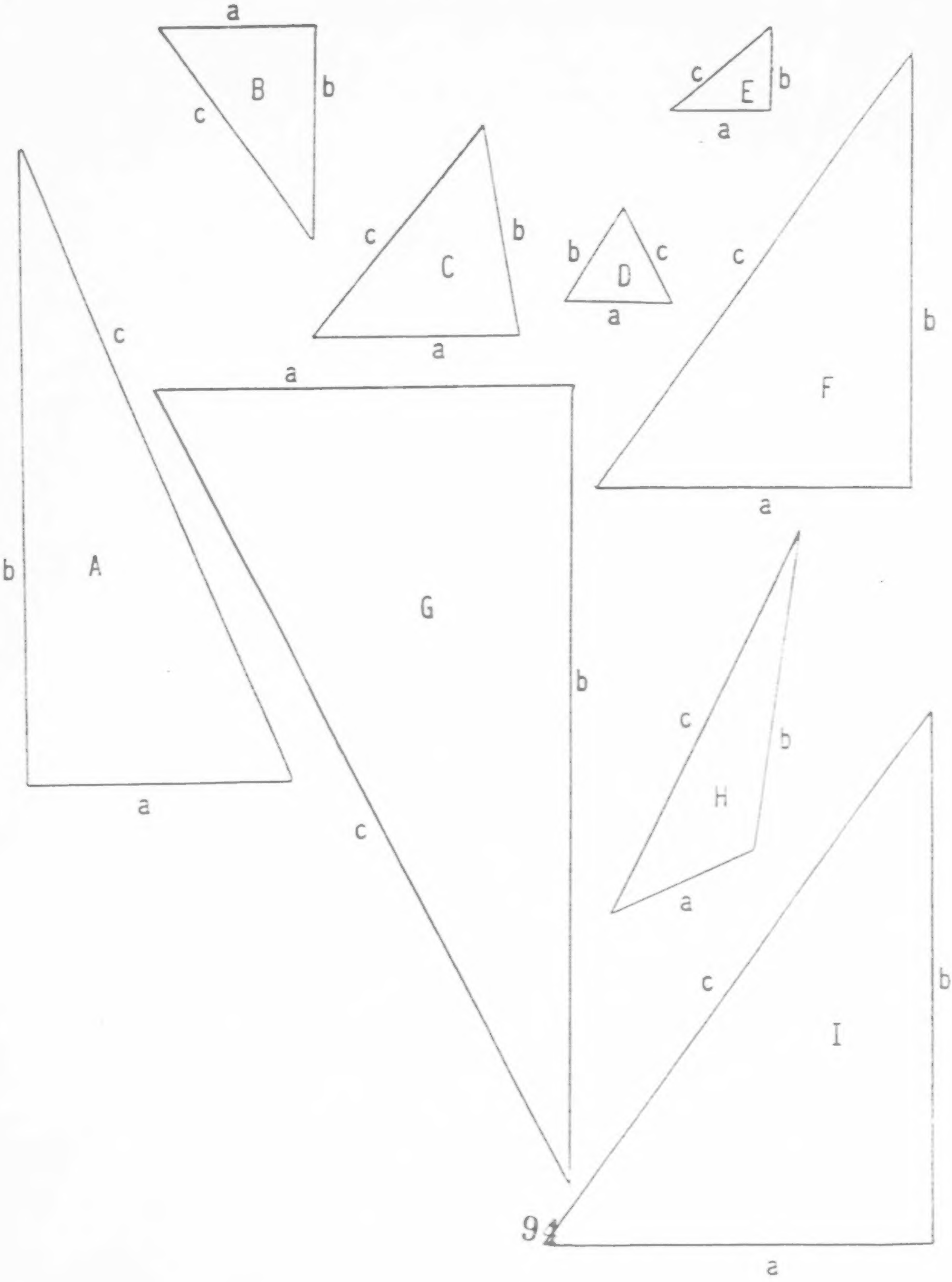
3. Study the results in the right side of the table. Look closely at the numbers that correspond with the right triangle. Write about any observations you've made.

4. Do you think a 9x10x11 centimeter triangle is a right triangle? First, make a prediction. Then check your prediction by making a drawing.

5. Is a 9x12x15 centimeter triangle a right triangle? First make a prediction. Then check your prediction by making a drawing.

THE AMAZING PYTHAGORAS

Activity Sheet 2.3F


a
B
b
c
c
E
b
a
c
C
b
a
b
D
c
a
c
F
b
a
c
A
b
a
a
G
b
c
c
H
b
a
c
I
b
a

THE AMAZING PYTHAGORAS

Activity Sheet 2.3F

The sequence of overhead transparencies needed for the informal proof of the Pythagorean Property follows.

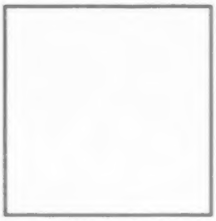


Fig. a

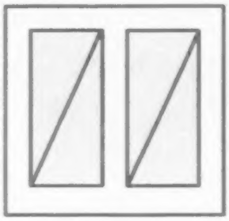


Fig. b

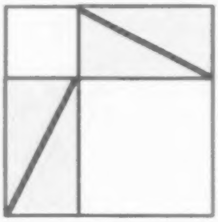


Fig. c

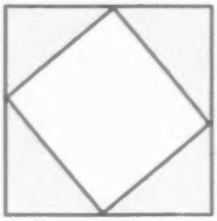


Fig. d

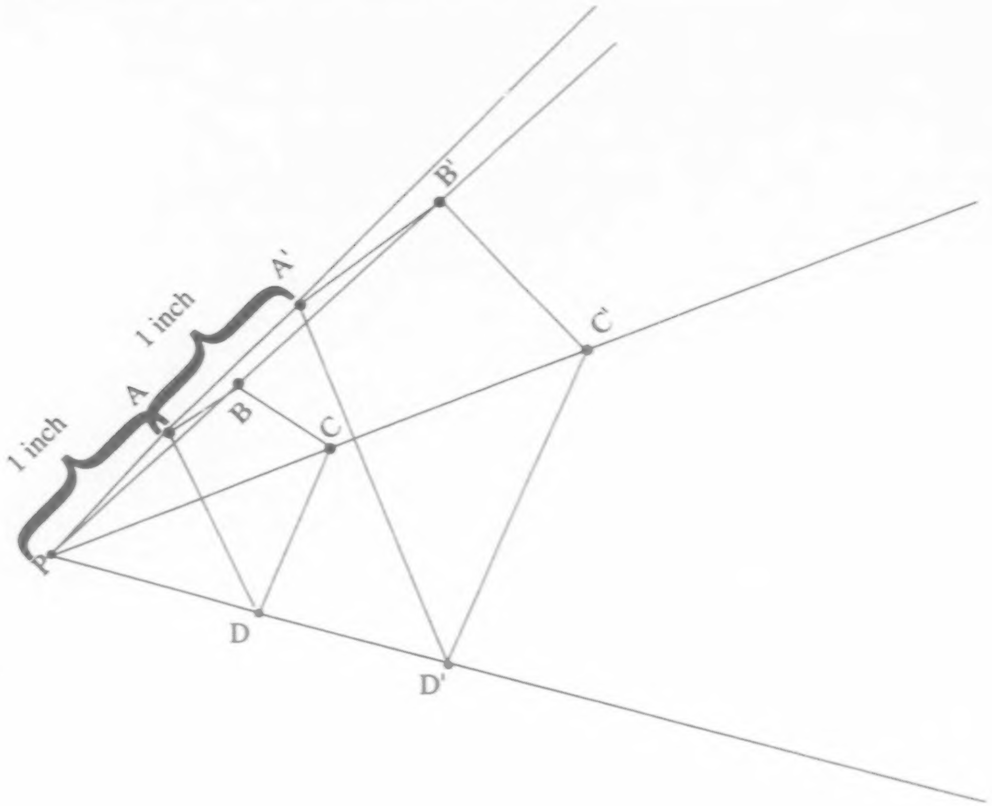
The areas of the light spaces in Figures b, c, and d are equal. In Figure c it is visually obvious that the "light space" area is $a^2 + b^2$; in Figure d, c^2. The formula for the light space in Figure b is not visually obvious but, of course, it could be expressed as $a^2 + b^2$ or as c^2.

Reproduce first, then cut here to use lower portion for overhead transparencies.

Enlarging Logos
Activity Sheet 2.4G

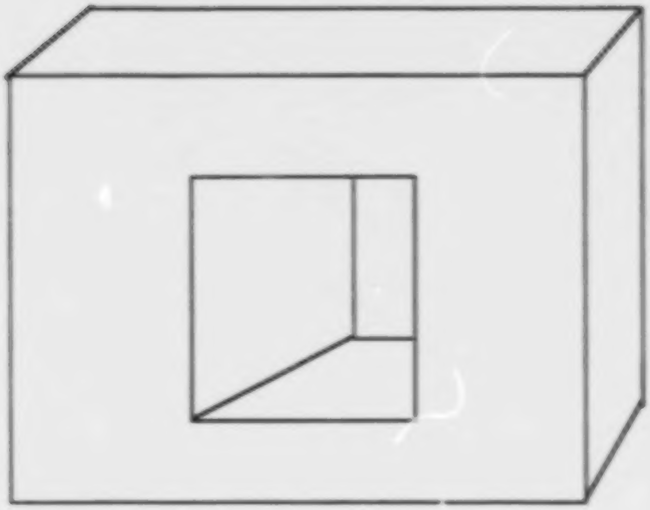
Often, in advertising, logos need to be enlarged.

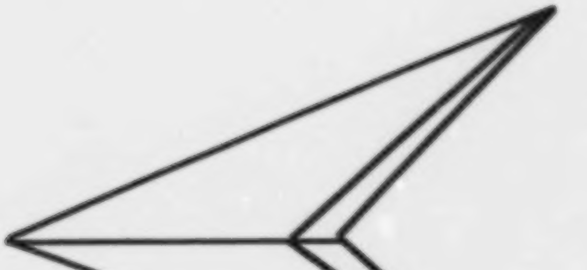
Take a simple drawing and enlarge it several times using a point of projection.
Choose a point of projection (P).
Draw lines from Point P through each vertex on the polygon extending out past the vertex.
Measure the distance from P to vertex A.
Mark off the same distance on the line.
Call it A' (A prime).
Repeat for each vertex.
Connect the vertices.
(Distances can be doubled, tripled, etc. for greater enlargements.)



QUICK DRAW

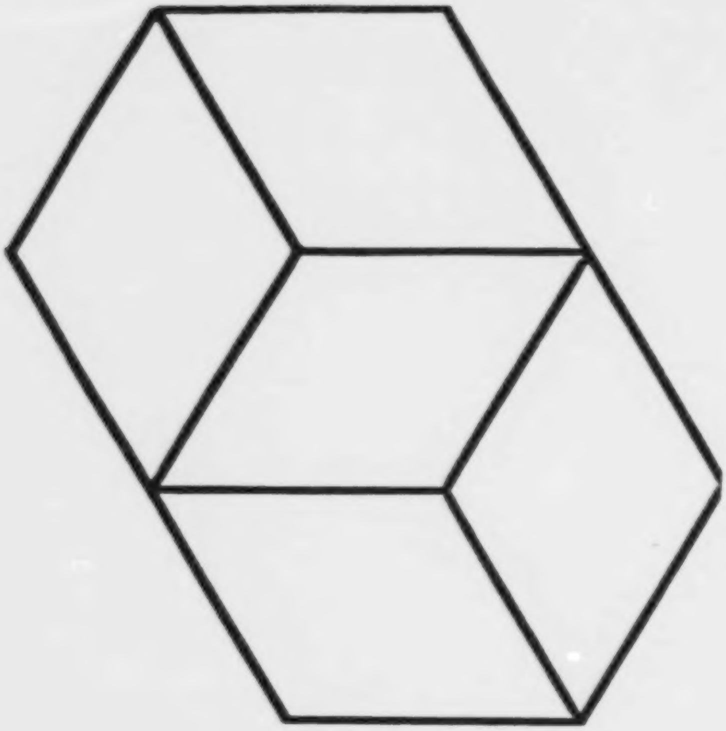
Activity Sheet 2.5 A

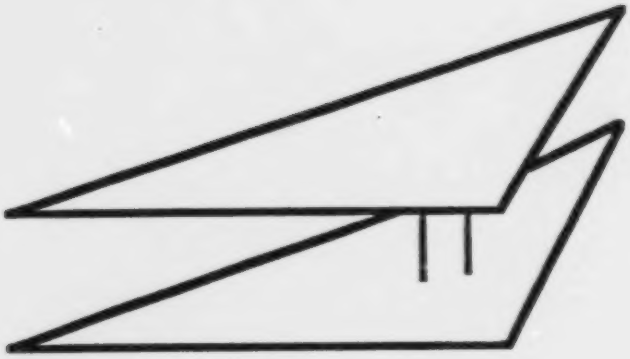




QUICK DRAW

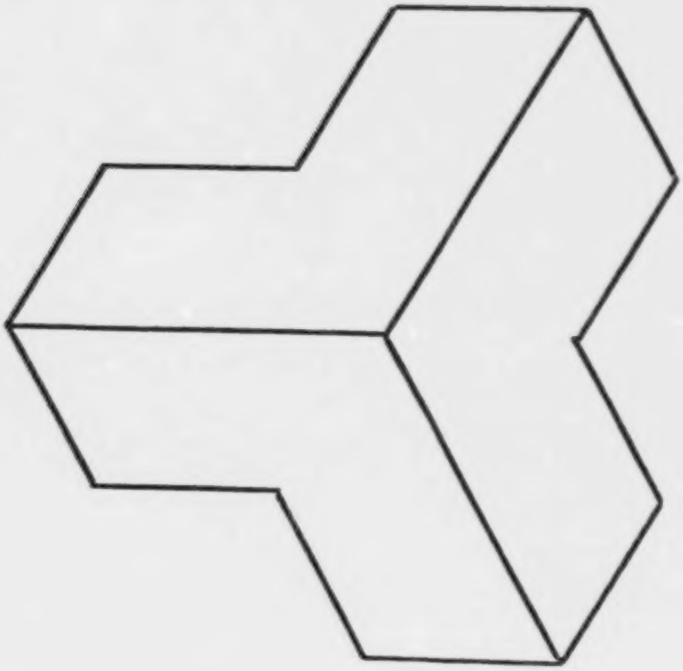
Activity Sheet 2.5 A





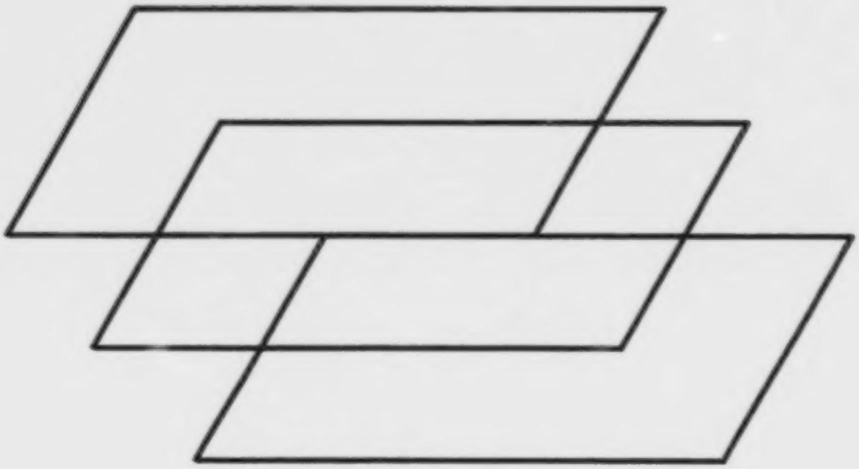
QUICK DRAW

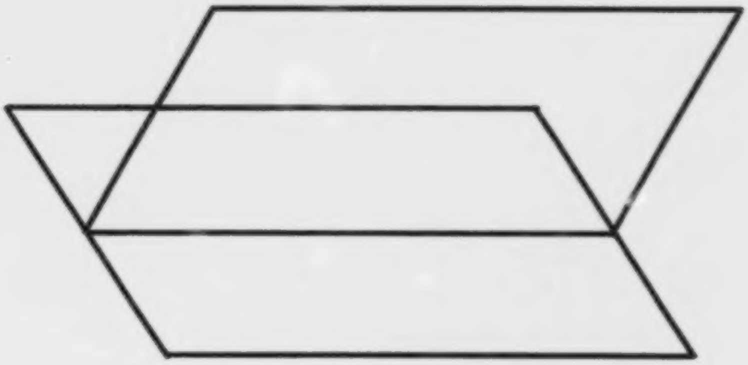
Activity Sheet 2.5 A



QUICK DRAW

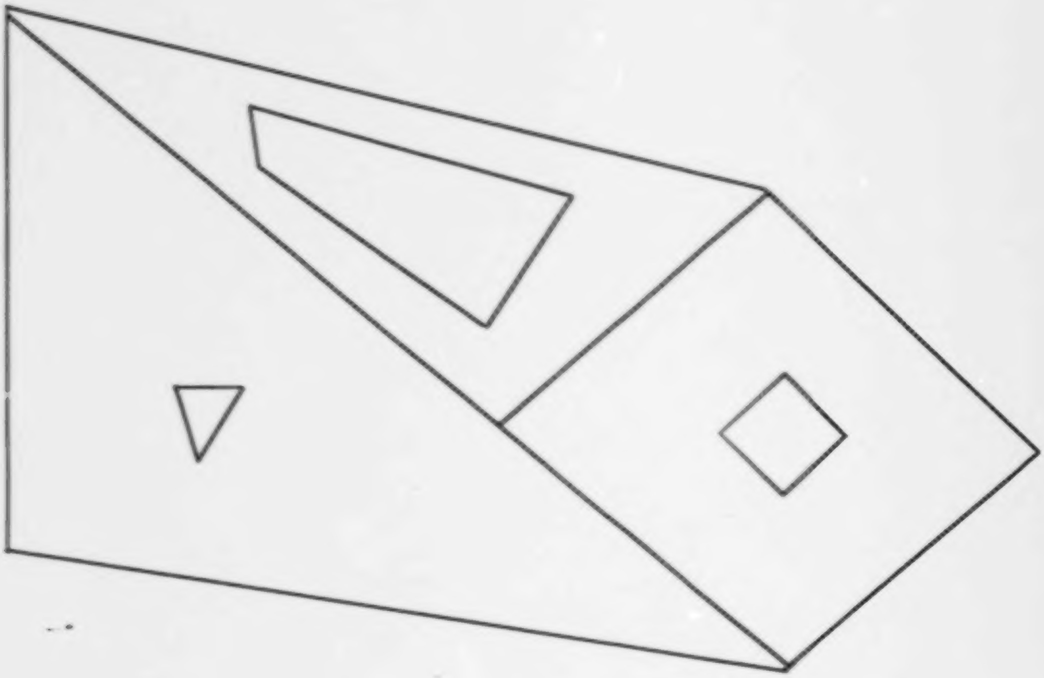
Activity Sheet 2.5A





QUICK DRAW

Activity Sheet 2.5A



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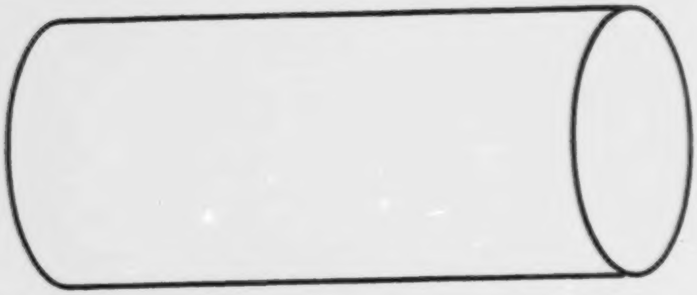
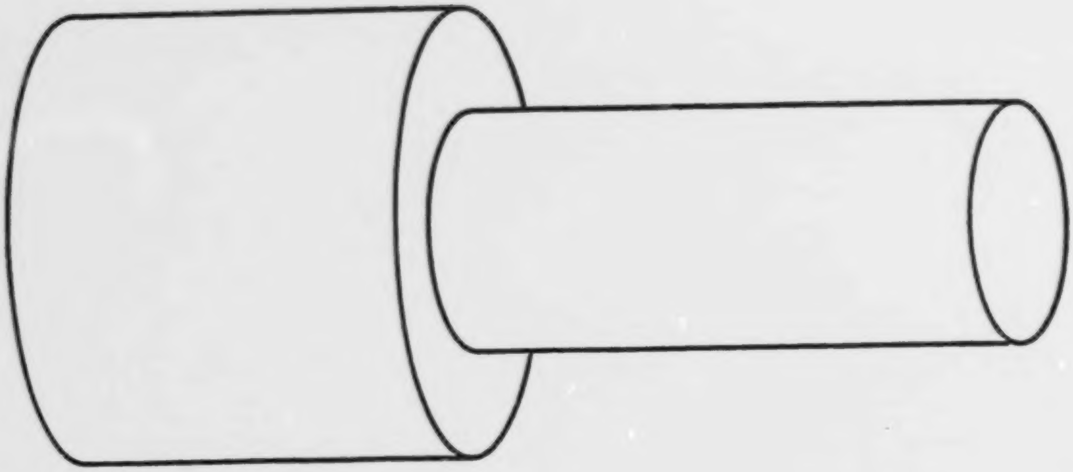
QUICK DRAW

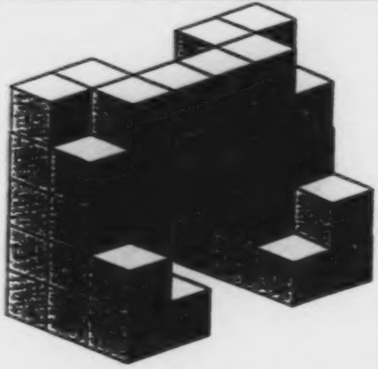
Activity Sheet 2.5 A





GRID AND GRAPH

Activity Sheet 2.5H



Going Up
Activity Sheet 2.51

Going Up # 1

There are nine blocks in all.

There are three colors used.

Going Up # 1

No two blocks of the same color touch.

Going Up # 1

The ends of the building are identical towers. Top blocks of both towers are red.

Going Up # 1

There are two red blocks on the bottom level.

Going Up # 1

All blue blocks are on the bottom level.

Going Up # 1

There are 2 yellow blocks.

Going Up # 2

There are seven blocks in all.

Two of the blocks are red.

Going Up # 2

No two blocks of the same color share a face.

Going Up # 2

The only block on the third level is green.

Going Up # 2

The blue blocks share an edge.

The red blocks share an edge.

Going Up # 2

Three different colored blocks form the bottom level.

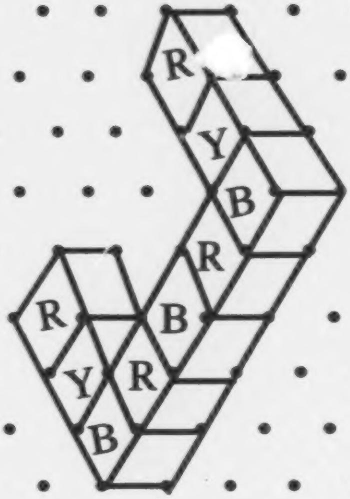
Going Up # 2

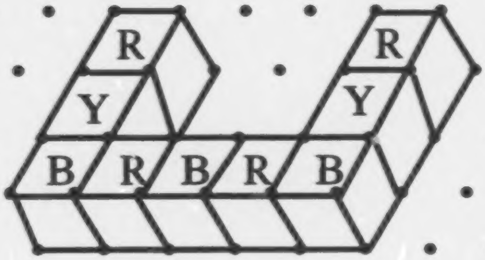
There are more green blocks than any other color.

GOING UP #1

Activity Sheet 2.5I

Possible Solutions

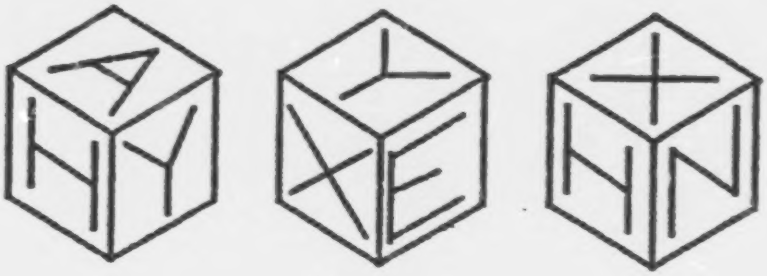




OPPOSITES ATTRACT

Activity Sheet 2.5J

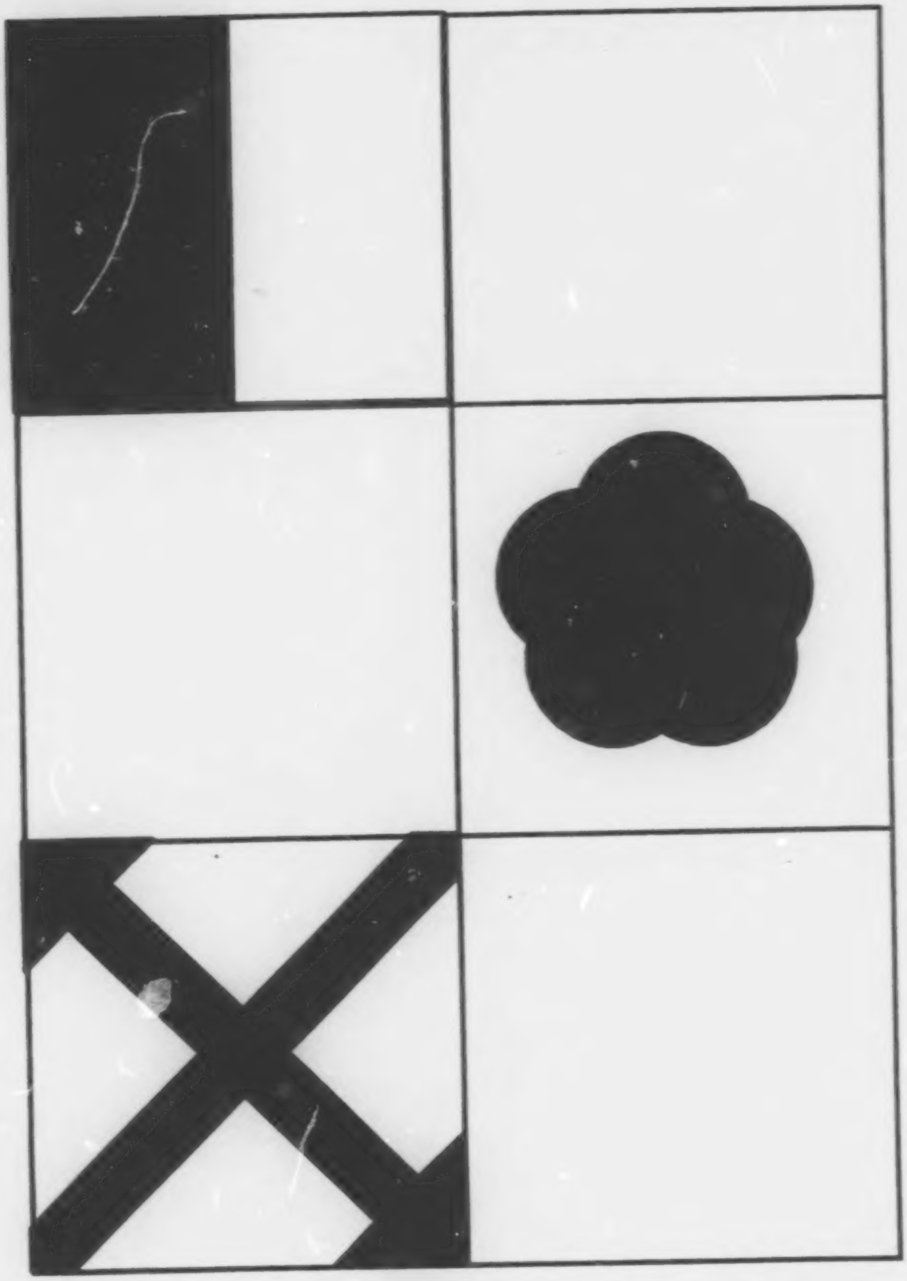
Below are three views of the same cube.

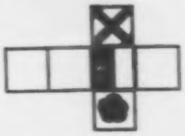


What letter is on the face opposite (1) H, (2) X, and (3) Y? (Give your answer in the same order.)

Solution:
From the first and third views, the letters on the faces adjacent to H are A, Y, X, N. Then the remaining letter E is opposite H. Similarly, from the second and third views, the letters on the faces adjacent to X are E, Y, H, N. Then the remaining letter A is opposite X. The letters which have not been paired are Y and N. They must be opposite each other.

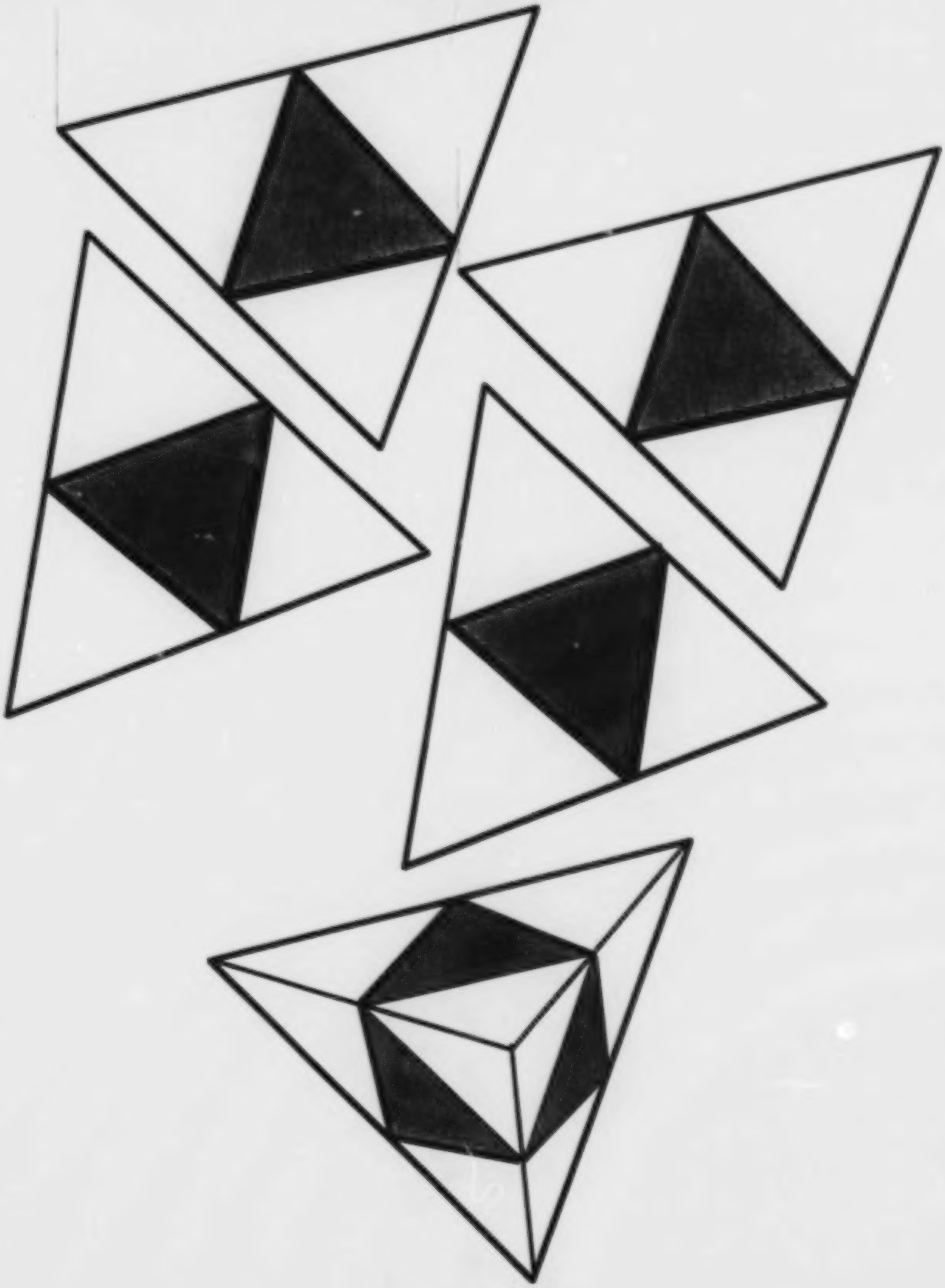
ACTIVITY SHEET 2.6C





THREE-DIMENSIONAL TRIANGULAR PYRAMIDS

Activity Sheet 2.6E



BUILD A MODEL

Activity Sheet 2.6F

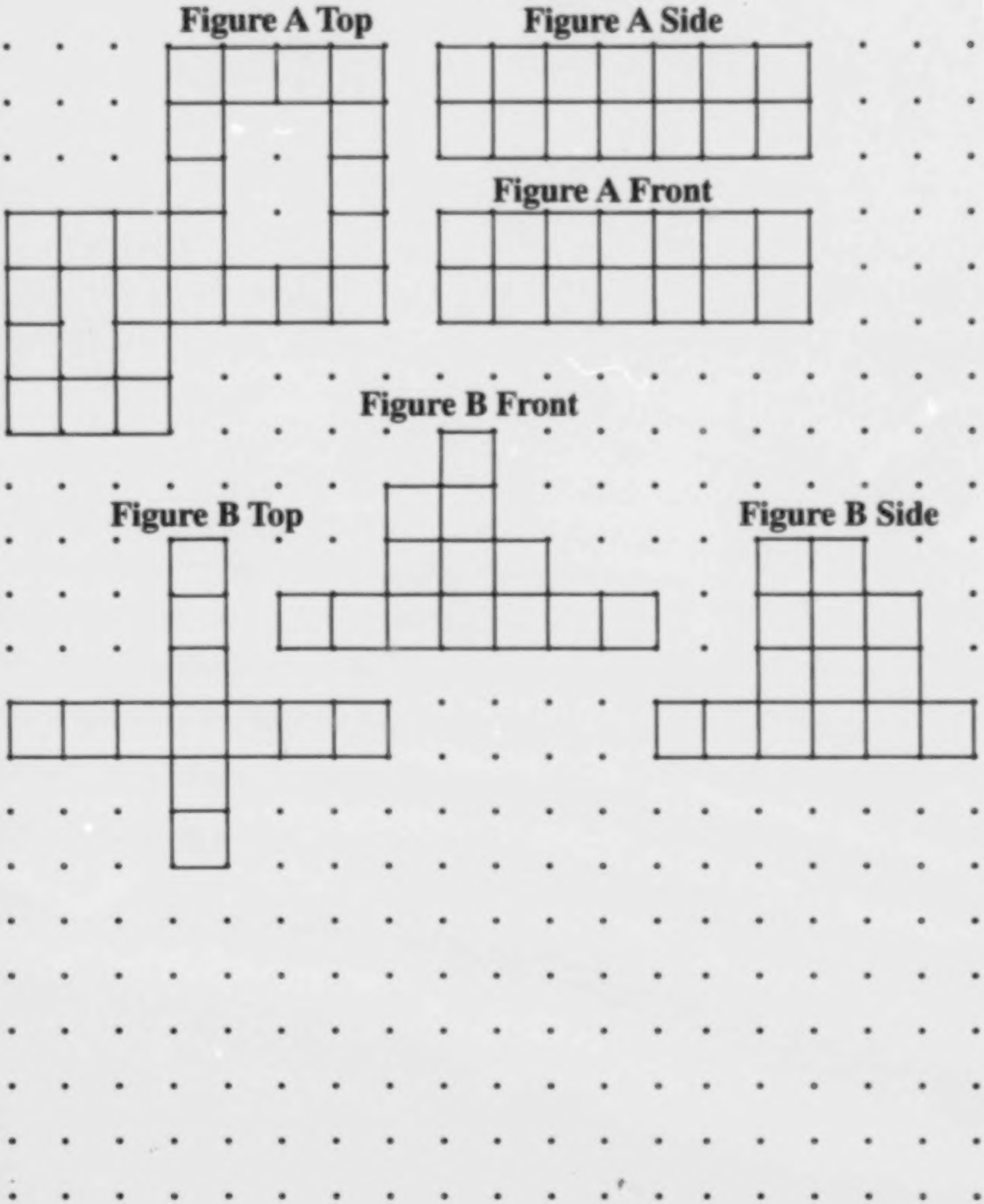


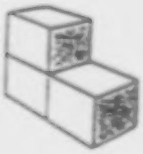
Figure A Top
Figure A Side
Figure A Front
Figure B Front
Figure B Top
Figure B Side

INTERLOCKING CUBES

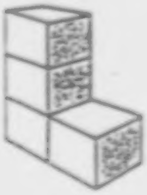
Activity Sheet 2.6H

The following solids can easily be made from 27 small wooden or plastic cubes. You may even try sugar cubes and some strong glue or rubber cement. Create the seven different combinations below by carefully glueing cubes together. No pieces are alike, and some are mirror images of each other. The solids can now be used to build the Interlocking Cube Models (see blackline master "Interlocking Cube Models").

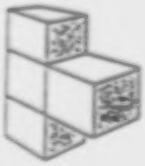
1.



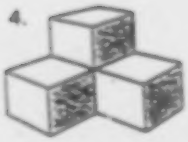
2.



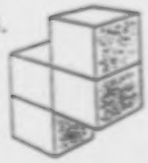
3.



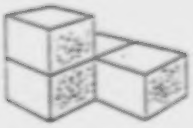
4.



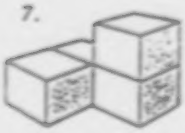
5.



6.



7.



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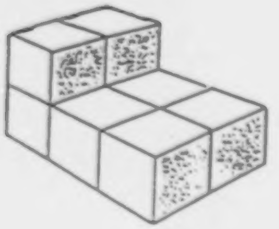
INTERLOCKING CUBE MODELS

Activity Sheet 2.6 H



1. Create the figure above by using only two Interlocking Cubes.

2. Create one large cube using **all** of the Interlocking Cubes (this can be done several different ways **and** it is a good idea to first place the cubes numbered 4, 6, and 7. Use piece #1 last.)

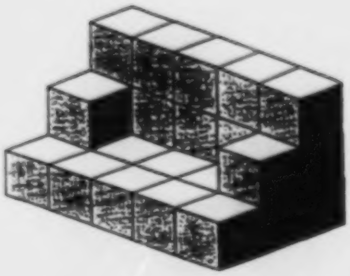
3. Using the following 10 models, recreate these with your Interlocking Cubes.

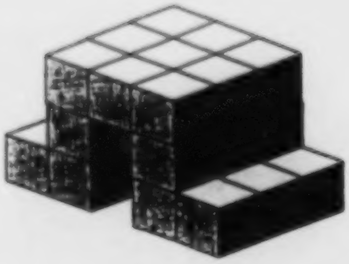
4. Create your own model and then draw it three-dimensionally.

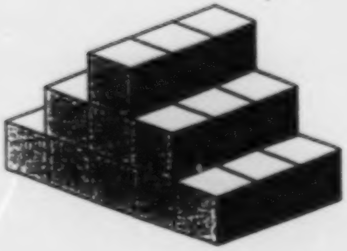
5. Exchange your three-dimensional drawing of your original model with a friend. Recreate each other's models from the three-dimensional drawings.

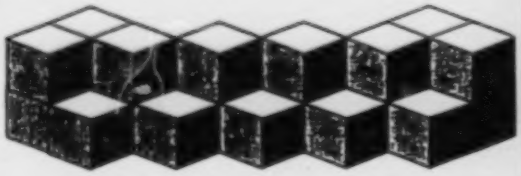
INTERLOCKING CUBE MODELS

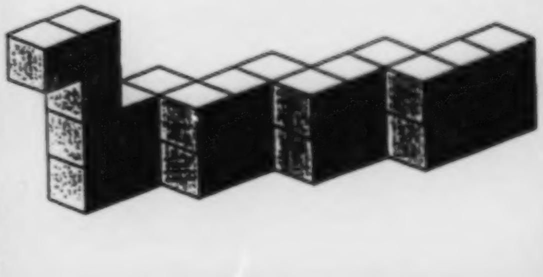
Activity Sheet 2.6 H

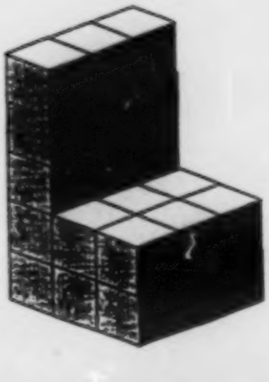






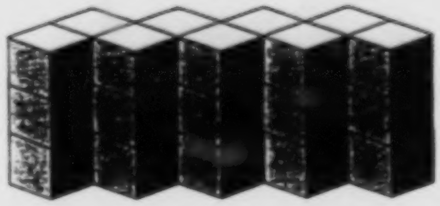


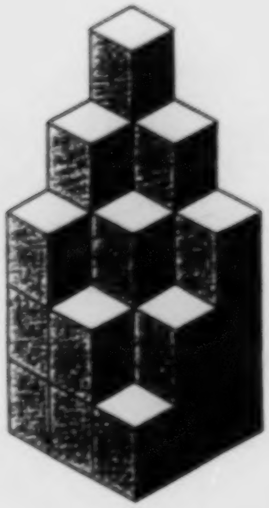


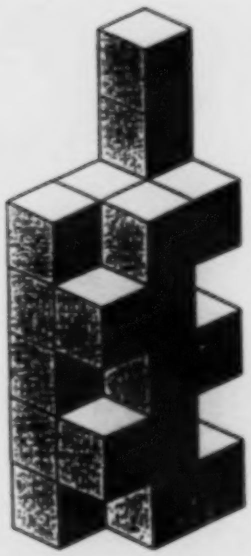


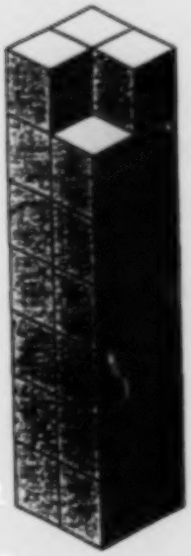
INTERLOCKING CUBE MODELS

Activity Sheet 2.6 H



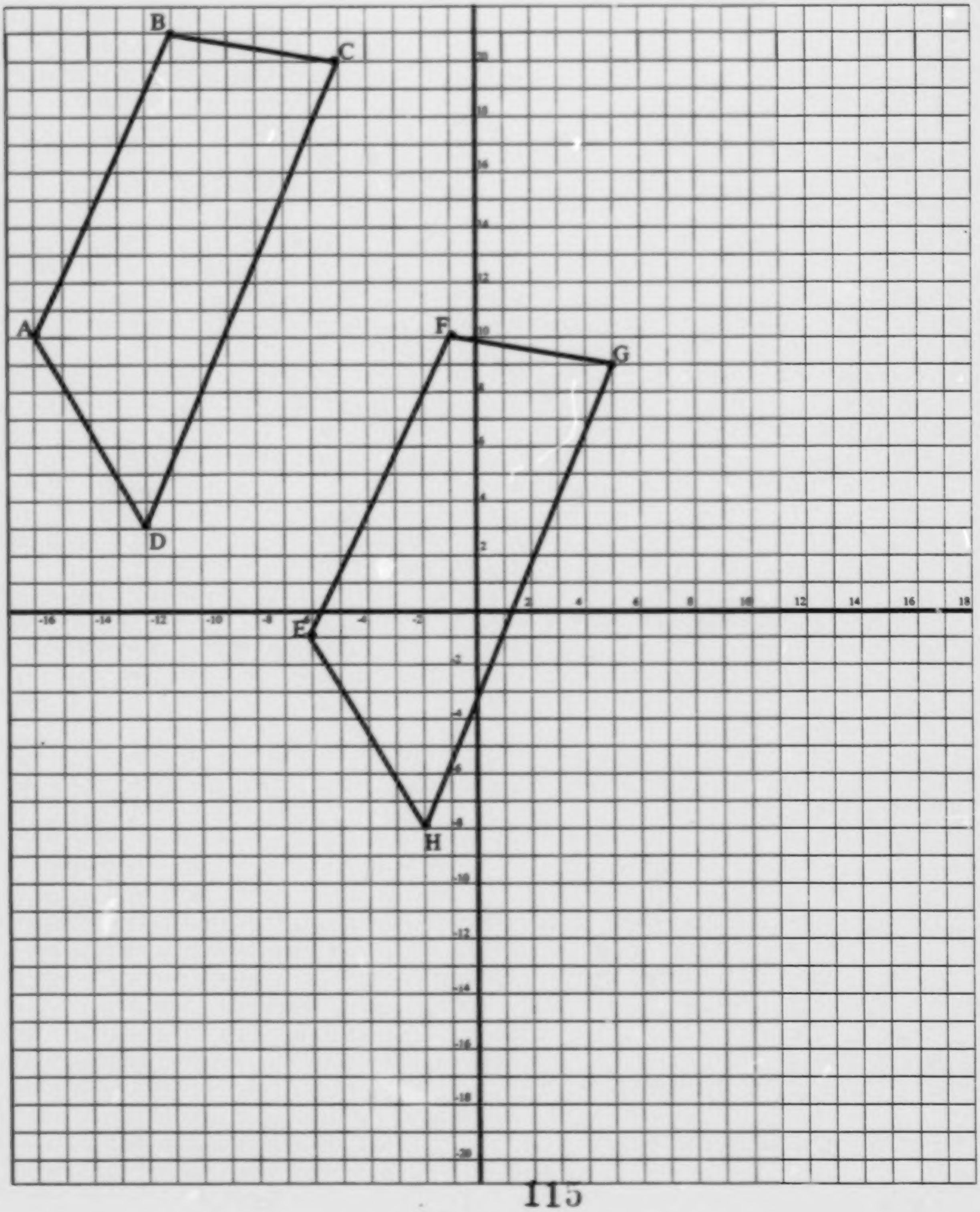






CONGRUENCY MATCH

Activity Sheet 2.7C



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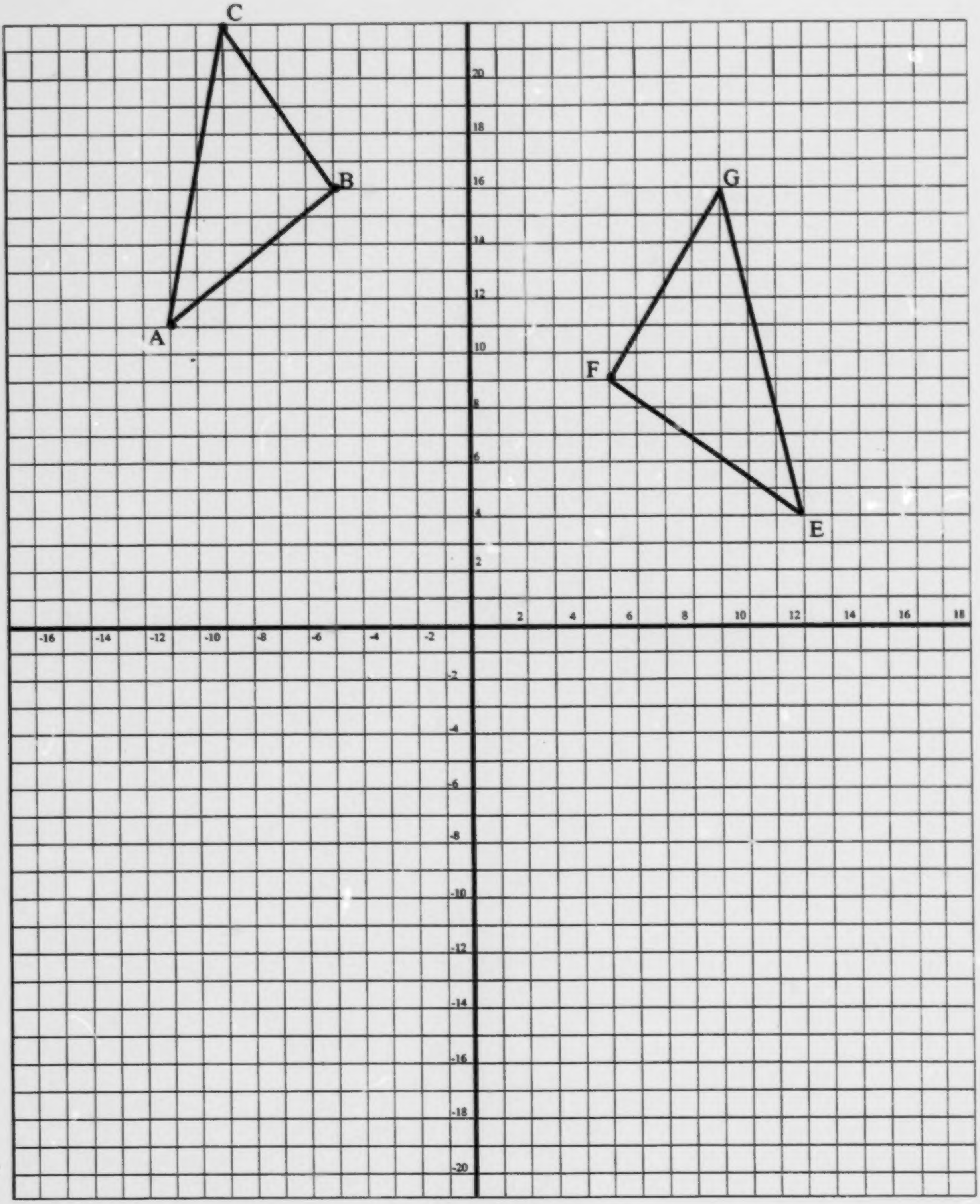
CONGRUENCY MATCH

Activity Sheet 2.7C



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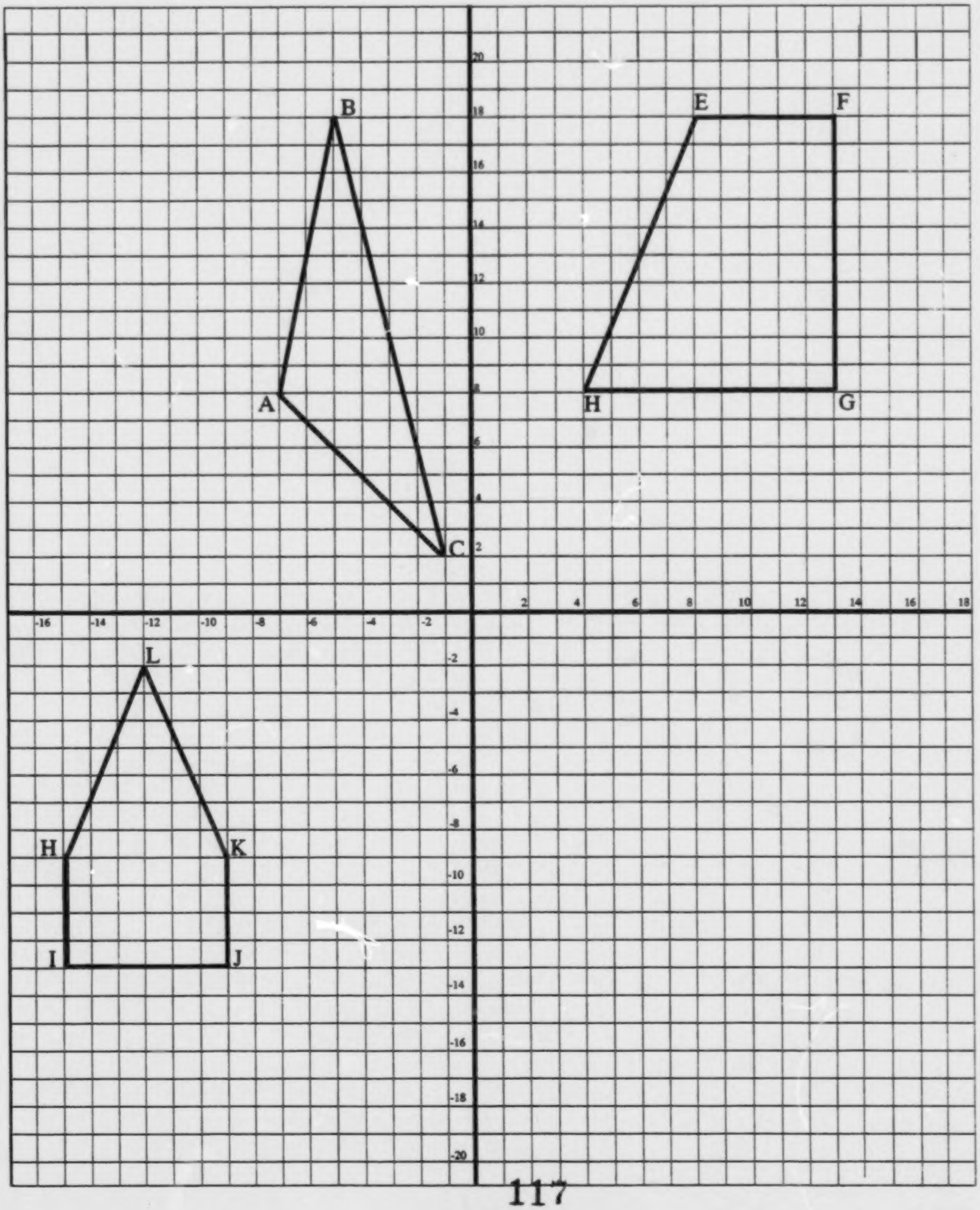
116

TRANSLATIONS & CONGRUENCY

Activity Sheet 2.7D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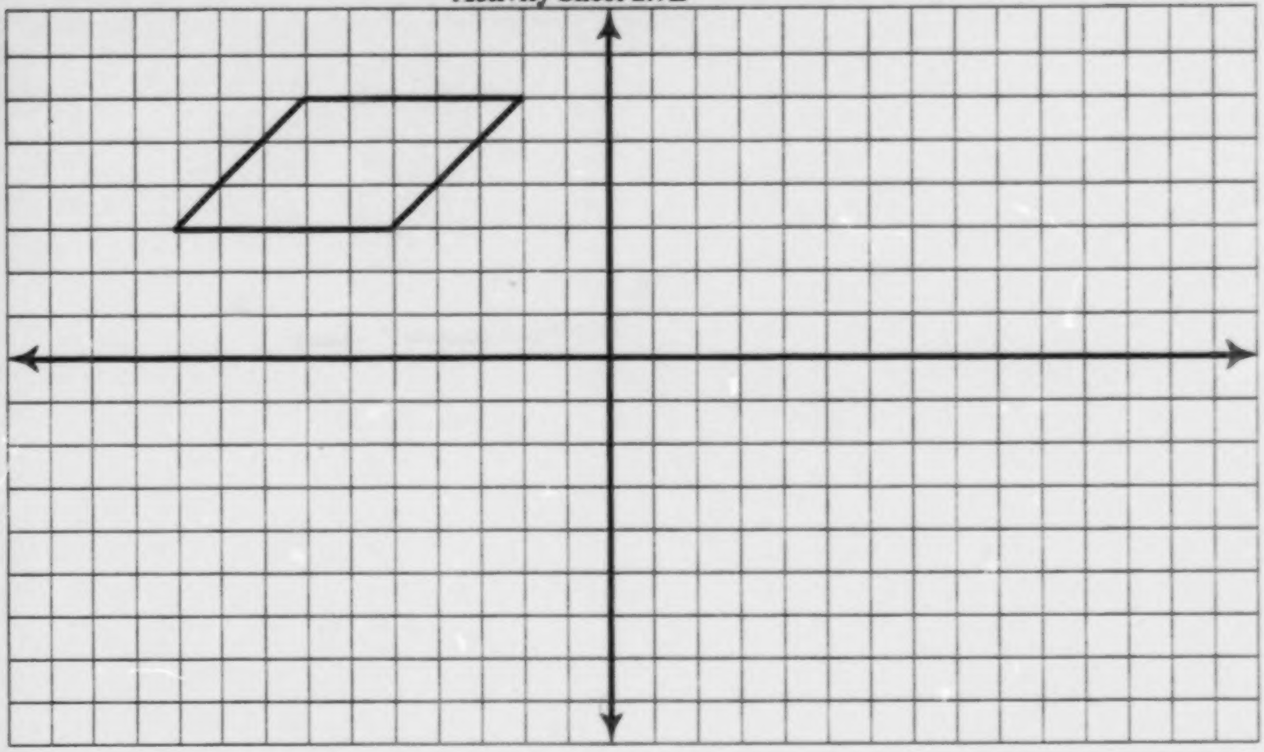
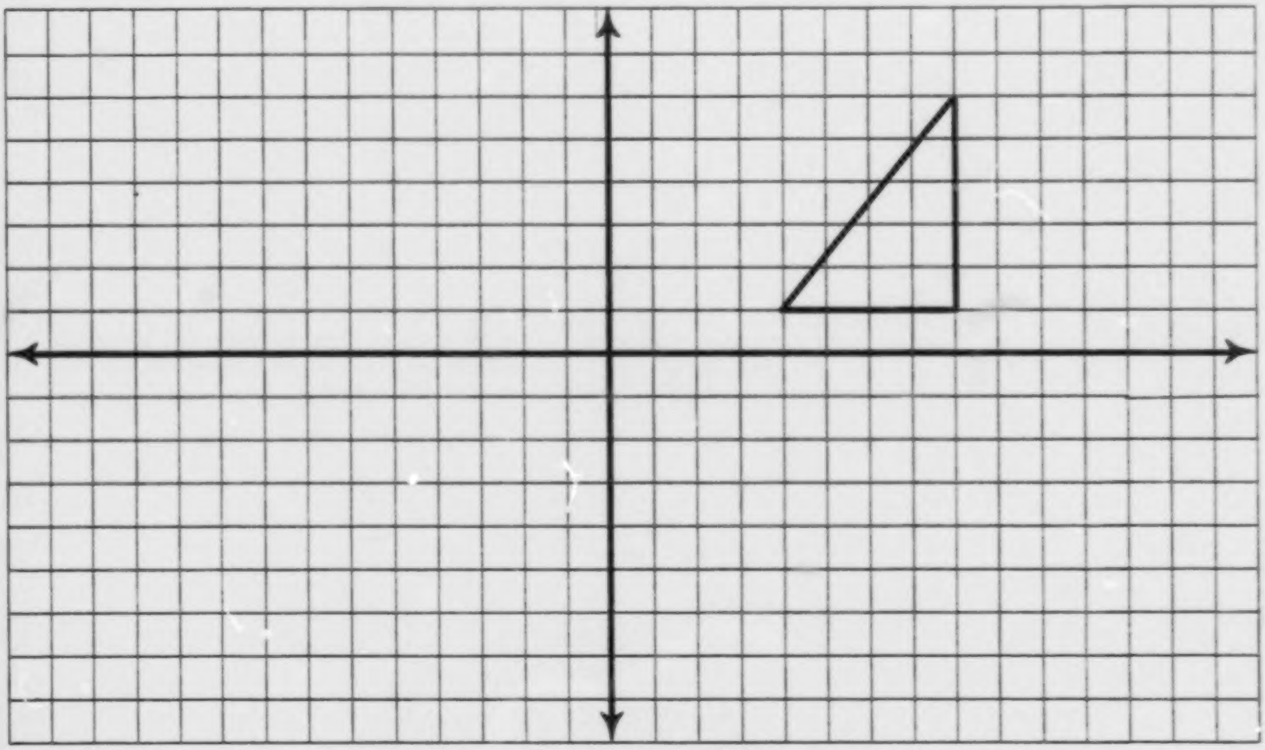
TRACING REFLECTIONS

Activity Sheet 2.7E

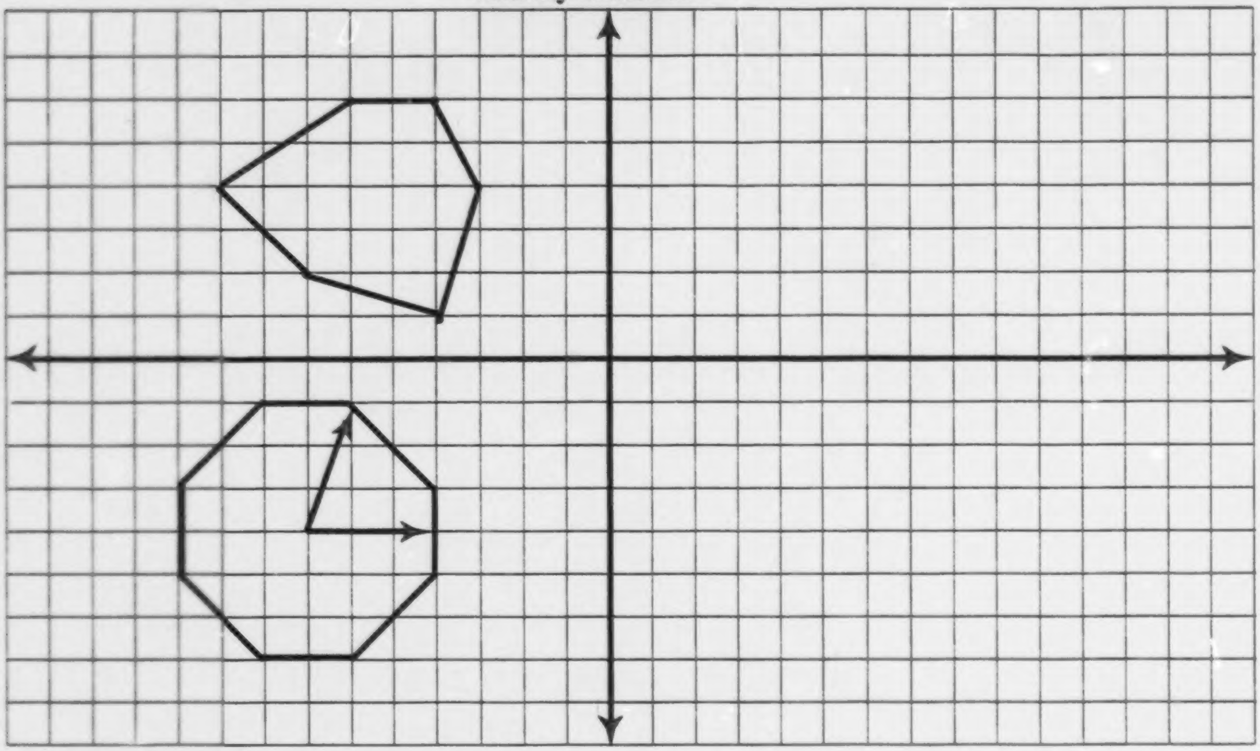


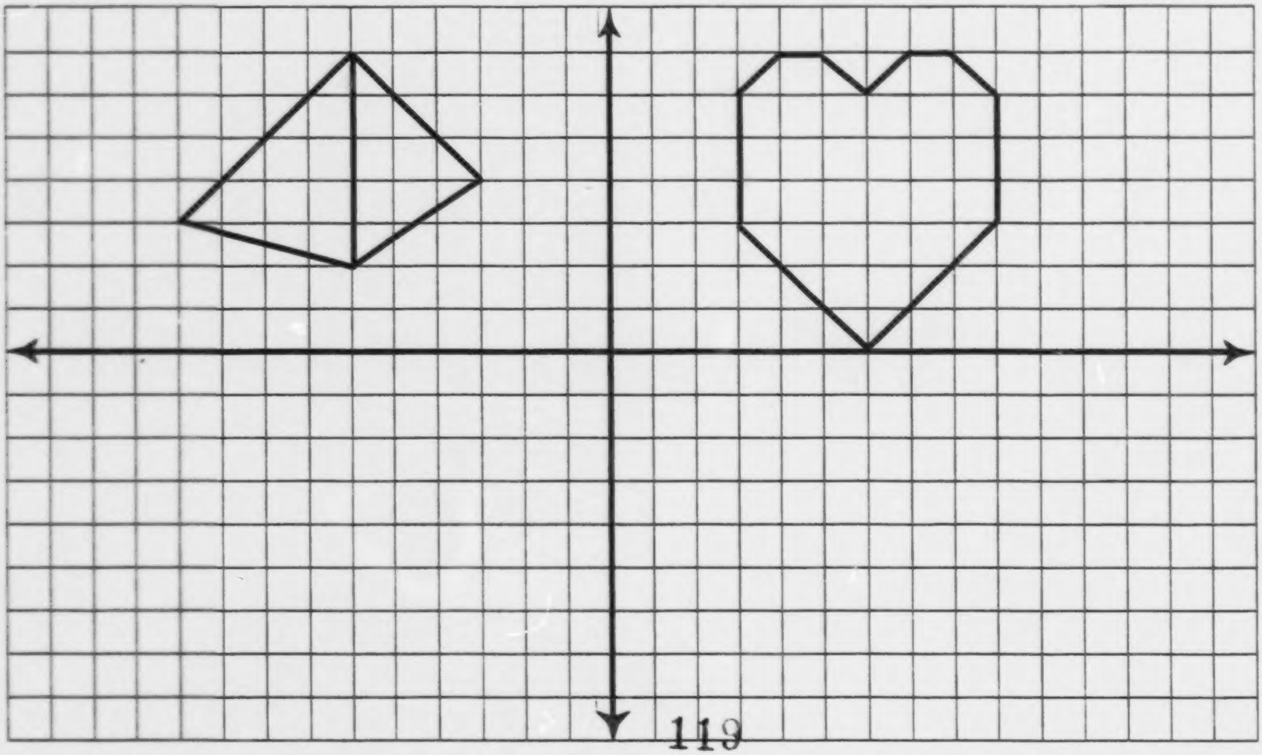


118

"MIRA" ACTIVITY

Activity Sheet 2.7F





CONSTRUCTING GEOMETRIC FIGURES

Activity Sheet 2.7 G

Directions:
1. With a piece of 1/4 inch graph paper mark and label the x and y axes.
2. Construct the figures described in the left column.
3. In the right column write the coordinates of the figures you constructed from the left column.

CONSTRUCT	COORDINATES
1. A square with a side of 2 units	
2. A rectangle with the dimensions of 2 by 5 units	
3. A square with a side of 6 units and one vertex at (1,2)	
4. How many different squares can be constructed meeting the requirements of #3? Construct them and write the coordinates of the vertices to the right.	
5. A rectangle with one vertex at (1,3) and dimensions of 2 by 3 units	
6. How many different rectangles can be constructed meeting the requirements of #5? Construct them and write the coordinates.	
7. Construct a square with one side having the endpoints (4, 5) and (4, 10).	
8. How many other squares can you construct that meet the requirements of #7? Construct them and write the coordinates.	
9. Construct all the right triangles you can with the vertex of the right angle at (3, 5) and the dimensions of the legs equaling 2 1/2 and 3 units.	

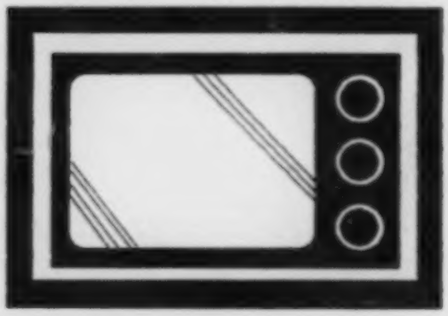
ISOMETRIC DOT PAPER

DOT PAPER

EIGHTH GRADE GEOMETRY STRATEGIES

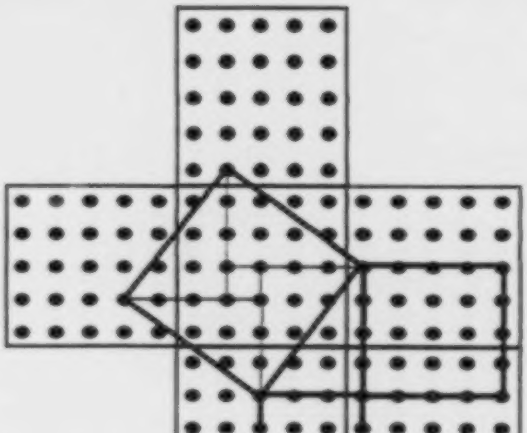
2.1 Use the Pythagorean theorem to find the missing side of a right triangle; use calculator when appropriate.

A. Have students discover the Pythagorean theorem through the use of deductive reasoning. Using the "Corner to Corner" sheet provided students find the maximum length of an object that will fit into a box. **See Activity Sheet 2.1A.** Students need to bring a box from home.



B. Use a newspaper, magazine, or television advertisement to introduce the concept of the Pythagorean theorem when shopping for TVs. Have students find all possible practical sizes. Remind students that each television screen is measured by the length of the diagonal of the screen. A spreadsheet would be perfect. **See Activity Sheet 2.1B.**

C. Use geoboards to reinforce the Pythagorean theorem and its converse and explore the relationship among the sides of a right triangle. Give groups of students six geoboards and three rubber bands of different colors. Let each rubber band represent a side of a right triangle with integral sides such as 3,4 and 5. Stretch the rubber bands to make squares on all sides of the triangle. Have students determine the Pythagorean relationship, $4^2 + 3^2 = 5^2$. Have them create different right triangles. Given two sides, have the students discover the third. (Regular dot paper can also be used.)



D. Have students work in small, cooperative groups to solve problems using the Pythagorean theorem.

Find the longest line segment that can be drawn in a rectangular prism that is 12 cm wide, 15 cm long, and 9 cm high.

Maria left her home and drove 15 miles north and then 20 miles west. How far from home did she travel?

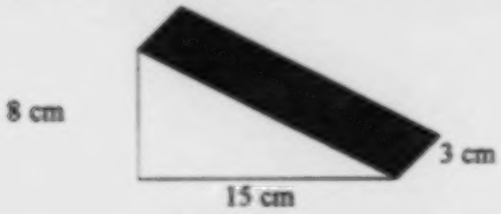
Try one of these BASIC computer programming activities and see if you can discover the answers using the Pythagorean theorem.

```
10 PRINT "ENTER LENGTHS OF
   LEG, HYPOTENUSE"
20 INPUT A,C
30 LET B = SQR(C^2-A^2)
40 PRINT "OTHER LEG IS ";B
```

```
10 PRINT "ENTER LEG, LEG"
20 INPUT A,B
30 LET C = SQR(A^2+B^2)
40 PRINT "HYPOTENUSE IS ";C
```

E. Have students work in small groups to draw several right triangles on rectangular graph paper. They should measure the hypotenuse and both legs. Have them confirm their measurements using the Pythagorean theorem.

F. Have students use the design below to find the area of the shaded face. You may extend this activity to geoblocks and ask for the area of a colored or labeled face.



G. Give students a set of three ordered pairs which have integer coefficients, and which form a right triangle on a rectangular coordinate grid. Ask students to find the lengths of all three sides of the triangle. (Note: Let a unit on the graph paper grid equal 1 unit.)

H. Use the Taxi Cab Geometry and City Maps activity sheets or have students draw a map of the streets in their town. Make streets intersect at right angles. Calculate the cross country distance from the beginning to the end of a taxi's trip. (Note: The taxi's travel should make a right angle, and the cross country distance is the hypotenuse.) **See Activity Sheets 2.1H.**

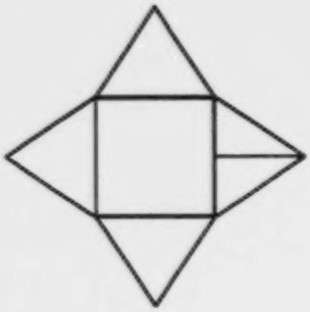
Extension: Use Pascal's Triangle to count the number of ways to get from one point to another on the map if you always move up or to the right.

I. Have students find Pythagorean triples by using the formulas indicated on **Activity Sheet 2.1I.**

J. Students' critical thinking and computer skills can be enhanced by using transformations of the Pythagorean theorem. Have students use a computer spreadsheet to record data for two sides of a right triangle, and use a formula to find the third side. **See Activity Sheet 2.1J.**

K. Use the Pythagorean theorem to solve real world problems. **See Activity Sheet 2.1K.**

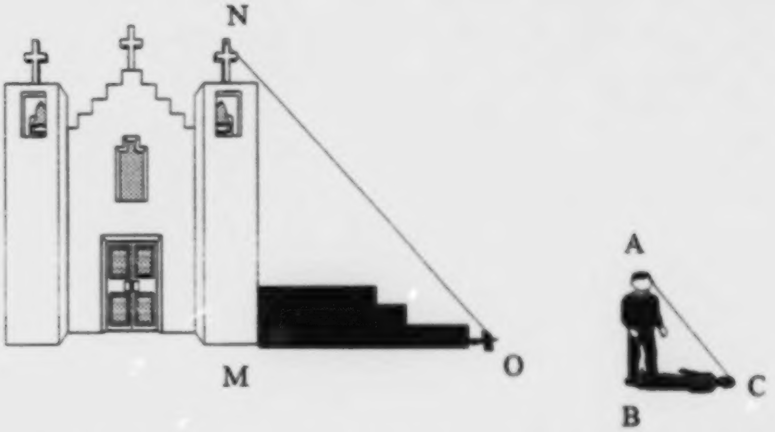
L. Earlier in this century, some people believed that the shape of the Egyptian pyramids had the ability to focus energy which could mummify animals, sharpen razor blades, improve the ability of seeds to sprout, preserve food, and improve the ability of humans to think. Have students build pyramids that are proportional to the Great Pyramid in Egypt. The ratio of the Great Pyramid's height to its base length is approximately 7:11. Have students make a pattern for a pyramid by drawing a square with an isosceles triangle attached to each side. If the square has a side length of 6 cm, what should the height of the finished pyramid model be? What should the height and leg length be on each of the isosceles triangles so that the pyramid will have the appropriate height?



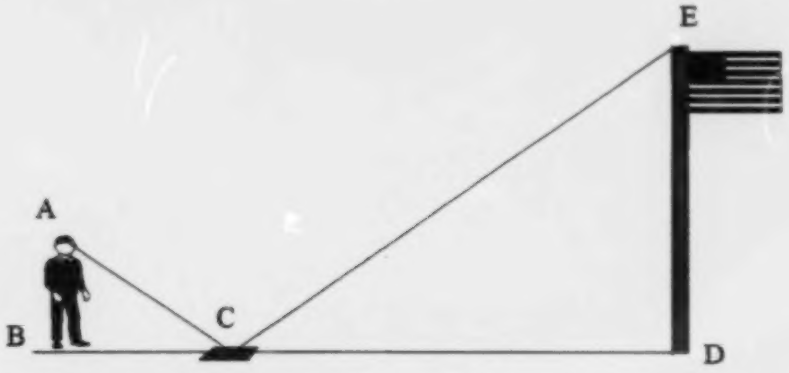
M. Make a Fold Drop & Pop box. Use the extension in the exercise to apply the Pythagorean theorem. **See Activity Sheet 2.1M.**

2.2 Solve problems related to similar figures using indirect measures to determine missing data.

A. Have groups of students apply the properties of similar triangles to indirect measurement by choosing some objects such as buildings, flagpoles, or trees to measure using the length of shadow the object casts.

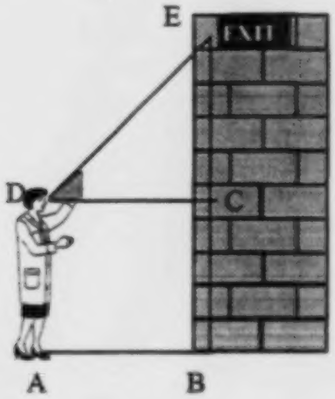


B. Give students a mirror with a point marking its center, a measuring tape, pencil, and calculator. Have them find the height of a flagpole using two similar polygons. **See Activity Sheets 2.2B**.



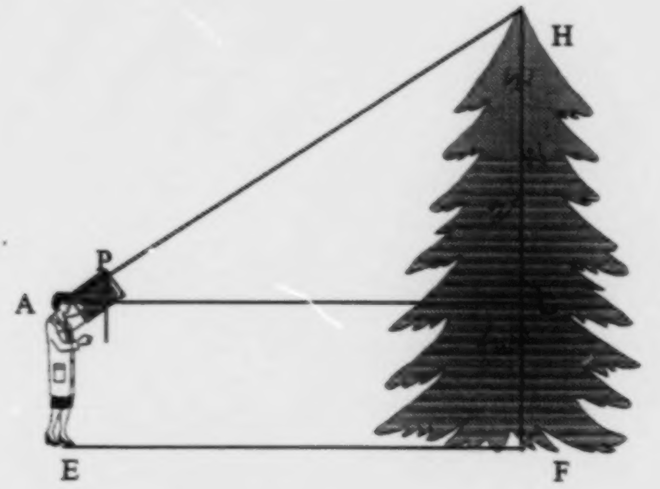
C. Give students a copy of "Carolina Capers" handout, rulers, and a calculator and then have them use ratio and proportion along with the concepts of similar figures to find missing measures. **See Activity Sheet 2.2C**.

D. Students may use the following instructions to measure heights indirectly. Fold a square piece of paper along a diagonal. Sight along the fold to the top of the object. Keep the bottom of the paper level with the ground. You may have to move closer to or further from the object. When you find the spot where you can sight according to these directions, make measurements and calculate the height of the object. See diagram.

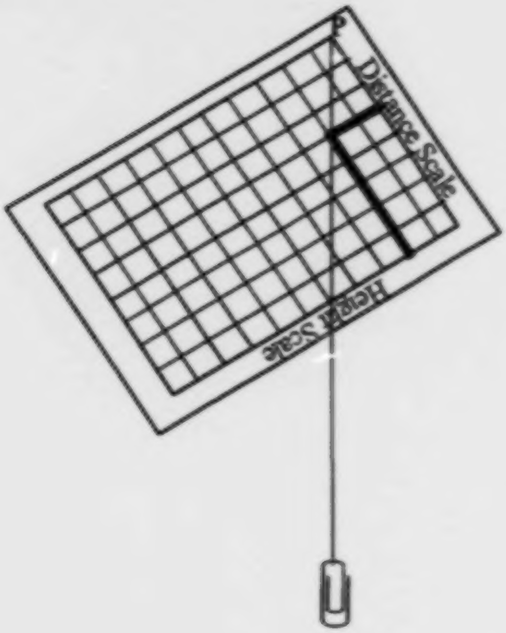


AB is the same distance as from the eye to point C. Since your triangle has two equal sides, triangle DCE also has two equal sides with DC = CE. The measurement of your eyes above the ground is the same as BC. You can now measure the height of the exit sign.

E. Students may measure heights indirectly using a hypsometer. Copy **Activity Sheet 2.2E** onto card stock. Cut out the view finders and fold up so that the viewer can sight through both holes. Attach a string at least 12 inches long at point P. Attach a weight to the other end of the string. Sight through the viewers to the top of the object to be measured.



Measure the distance from where the person sighting with the hypsometer is standing to the base of the object (EF). EF is the same distance as AG. For the person sighting, measure the distance from the eye to the ground, AE (same as GF). While keeping the hypsometer sighted at the top of the object, a second student locates the distance of EF on the distance scale. From that point on the distance scale, trace along the line perpendicular to the distance scale until it intersects the string. There is a perpendicular line from the height scale that also intersects at the same point. Add the measure from the height scale to AE to determine the height of the object.



F. Explore the use of computer software with students. Computer software such as the "Geometer's Sketchpad" provides a variety of opportunities for the exploration of characteristics and properties of two- and three- dimensional figures including those of similar figures. They allow students the flexibility of being able to manipulate large figures, and position them where desired.

2.3 Draw 3-dimensional figures from different perspectives (top, side, and front).

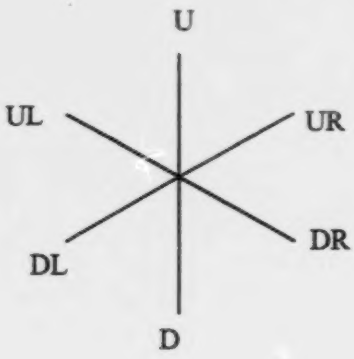
A. Have students make a 3x3x5 inch rectangular prism from card stock. Let them color each face a different color. Draw this prism from different views. If this were a solid cube with the outside painted and we cut it into 1 inch cubes, how many would there be? How many would have paint on four faces? three faces? two faces? one face? no face?

B. Using various geoblocks, have students trace the faces of the geoblocks and create orthographic drawings and buildings. Then give them an orthographic drawing and have them build the model. **See Activity Sheets 2.3B.**

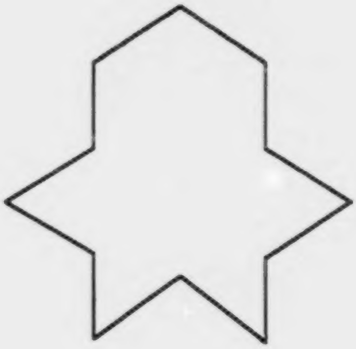
C. Using isometric dot paper and the code below, have students construct diagrams like the one pictured below. Explain how isometric dot paper is used with the various code moves.

U = up
UR = up to the right
UL = up to the left

D = down
DR = down to the right
DL = down to the left



Have students draw this: (Pick any point to start). U, UR, DR, D, DR, DL, D, UL, DL, U, UL, UR.
It should look like this:



D. Have students use cubes to make up to six three-dimensional designs. Each student will write the directions for building his/her design. The directions are given to a partner who tries to reconstruct the building from the directions.

E. Using isometric dot paper, students can attempt to make drawings of the designs they built in D. above. Give the drawings to another student who will use cubes to reconstruct the building from the drawing.

Extension: Have the class attempt to make drawings of the cube buildings from different points of view. Try using letters of the alphabet and create them in 3-D.

F. Have students build three dimensional figures using a variety of objects. Have them identify the type and number of objects used and identify all of the faces of the figure. **See Activity Sheet 2.3F**.

G. Using geoblocks or other polyhedral models, generate drawings of the same shape (similar) based on projections of the model onto the chalkboard using the overhead projector. Show students the shadow of each face of the model by turning it while students draw a 3-d rendering of the model at their seats. Have students compare their drawings to the actual figures. If students have drawn a different figure, discuss whether or not their figure could have been formed with the faces which were displayed.

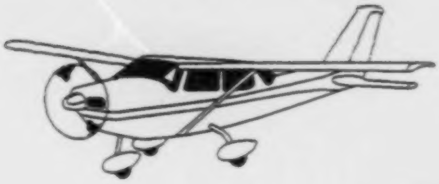
H. Students use the "Build This" activity to develop visualization skills as they study relationships among attributes of geometric figures. Have them name and write a description of each solid figure. **See Activity Sheet 2.3H**.

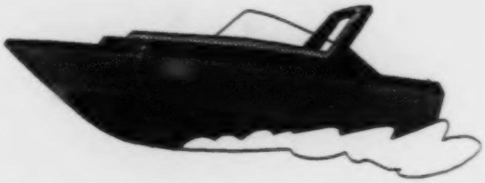
I. Students identify and draw models of three-dimensional objects found at school to reinforce spatial visualization skills and increase their ability to identify and find applications of geometry in the environment. Have groups of students write riddles to describe the figures and share them with the class or give to other groups to solve.

J. Enhance students' visualization skills as they explore geometric shapes. Using geoblocks, students select different shapes, put them together in a building, and then make a three-dimensional drawing of the shape from various points of view. Extend the activity by having students write descriptions of shapes and buildings to exchange. After exchanging, have students reconstruct the shapes from the descriptions.

K. Three-dimensional drawings can be generated using Logo software by writing directions for the Turtle to follow. Students can then print and compare their creations and display them on "Turtle Shells" in the classroom or hallway.

L. Grids of various sizes can be used to generate similar figures. Using artistic drawings of planes and boats, have students transfer similar designs from one grid size to another. **See Activity Sheets 2.3L.**





M. Have students find pictures of tetrahedrons, octahedrons, dodecahedrons and other space figures. Have them make patterns and construct these figures.

2.4 Graph on a coordinate plane similar figures, reflections, and translations.

A. Using their bodies, students can demonstrate rotations and translations. Stand in front of the room. Tell the students to imagine that the room is a clock and you are standing at the number twelve. Verbally give the students directions similar to the ones below. This activity quickly allows you to physically see students who do not understand rotation, clockwise, counterclockwise, and/or 90 degree turns. At first, it is easier to have students turn to face you before the next degree turn is announced.

Sample directions: 1) 90° clockwise turn
2) 90° counterclockwise turn
3) 180° clockwise turn
4) 180° counterclockwise turn

After students master these turns, have them turn from the point at which they are standing after the last turn.

B. **Use Activity Sheets 2.4B**. Students' visualization and spatial skills can be developed as they explore reflections and translations.

C. Give each student a copy of the "Let Your Little Light Shine," **Activity Sheet 2.4C**. Have them list the ordered pairs for each labeled point on the graph. Discuss and decide on a rule to reduce the lighthouse (i.e. divide each coordinate by 3). List the new ordered pairs and then draw the smaller light house on the same sheet or on a new sheet of graph paper.

D. Concepts of how figures are created on a coordinate plane can be introduced by giving students a set of points which when connected would make a polygon. Have the students plot the points on a coordinate plane. Connect the points in order and connect the last point to the first one. Write the name of the polygon formed.

Examples: A(0,2); B(3,2); C(3,0); D(0,0) rectangle
A(3,7) B(9,7); C(9,3); D(3,3) rectangle
A(3,1); B(1,3); C(-2,3); D(-3,-1); E(-1,-3); F(2,-3) hexagon

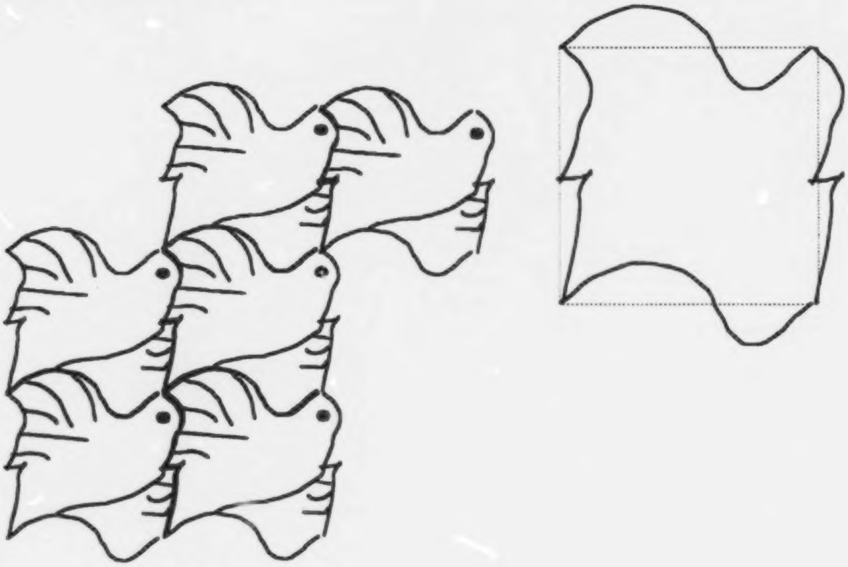
E. Using an overhead projector and a transparency of a coordinate grid with colored overlays of various polygons, demonstrate the concept of translation by sliding the polygons from one position to another. Let students carry out this action at their seats using a sheet of graph paper and a cut-out of a polygon.

F. Using cutouts of polygons and graph paper, students trace around the shape of a polygon to show its original location and then move it to a new location on the coordinate plane and trace around it once more to show the final location.

G. Using various patterns of wallpaper and clothing, students work in groups to write how translations and reflections could be used to make the pattern.

H. Working in groups, students draw a shape on a coordinate grid and give instructions to group members on how to draw three reflections or translations of the shape. Have groups display their pattern on a bulletin board for others to see.

I. Working from a square, students can form a tessellating pattern like the one shown. To do this, draw any variation of the top edge and translate it to the bottom edge. Draw a variation of the left edge and translate it to the right edge. Enlarge the pattern and decorate it to represent an object of some kind. Translate the pattern to form a tessellation.



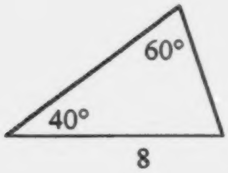
J. Have students explore the context of transformation of shapes drawn on a coordinate plane using the "Transformations in the Coordinate Plane" activity. See **Activity Sheets 2.4J**.

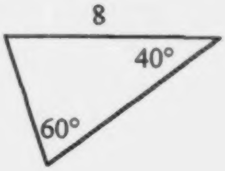
2.5 Explore the triangle congruency relationships: ASA, SSS, SAS.

A. Have students explore congruent triangles using straws to form triangles. Make transparencies to verify triangle congruency. **See Activity Sheet 2.5A**.

B. Use computer software to create congruent triangles given the measure of two angles and the length of the included side (ASA); given the length of two sides and the measure of the included angle (SAS); or the lengths of three sides (SSS) that are reflections or translations of each other.

C. Have students determine which corresponding parts will yield a triangle congruency using the "Triangle Congruency Practice" activity. Reinforce the definitions of triangles and their classifications according to the side and angle relationships of the figures by having groups of students share information on the measurements. Then they should sort triangles into various groups. **See Activity Sheet 2.5C.**





D. Reinforce the construction of congruent triangles by having students use a 3x3 portion of a geoboard to construct as many congruent triangles as possible based on ASA, SSS, and SAS relationships.

E. Make a set of cards showing pairs of congruent triangles. Shuffle and stack so students take turns drawing from this stack. They must identify the type of triangle and name the corresponding parts.

F. Use D-Sticks (colored dowels of different lengths with rubber connectors) to illustrate SAS, SSS, and ASA.

SSS -- Make a triangle from 3 different colored sticks. Give the same sticks to a student and ask him/her to make a triangle. Show that they are congruent.

SAS --Choose two dowels. Give someone else the same colors. Show that once you have chosen the angle between the two sides, the 3rd side is also determined.

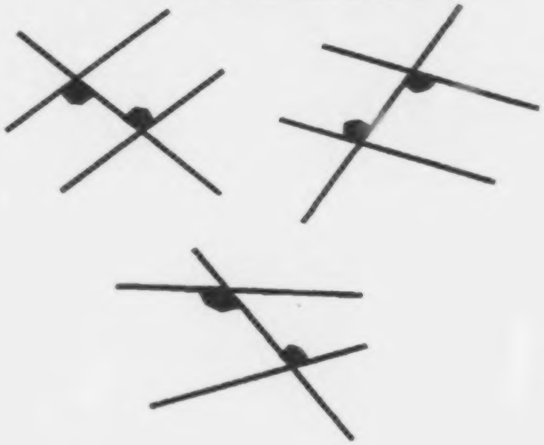
ASA -- Attach two long dowels to the end of a shorter dowel. Show that when you decide on the angles at the ends of the short dowel, the point of intersection of the two long ones is fixed.

2.6 Explore the relationships of the angles formed by cutting parallel lines by a transversal.

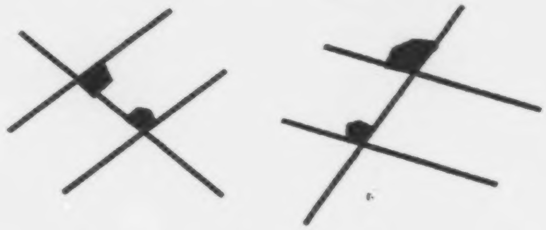
A. Using "Ladders and Saws," have the students explore concepts related to parallel lines and polygons. **See Activity Sheets 2.6A.**

B. To Be or Not To Be.
Develop the definition of alternate interior angles, alternate exterior angles, and corresponding angles through the use of a definition development card. Cards may be developed similar to the following example.

These are alternate interior angles.



These are NOT alternate interior angles.



Draw a pair of alternate interior angles.

What are alternate interior angles?

C. Students' visual skills can be enhanced and basic geometry constructions can be reinforced using everyday household items such as waxed paper or patty paper.

Example:
Construct parallel lines on a piece of waxed paper by making folds.

Construct a transversal by making a third fold across the parallel lines.

Students may then explore for angles which are congruent by folding the waxed paper so that angles are on top of each other in a variety of positions.

Once the angles are found to be congruent, have students draw a diagram showing which angles they found to be congruent.

D. Have students develop maps to show their routes to school. They can use street maps or the maps published by public transportation companies for reference. Have them identify streets that are transversal and parallel. Also have them identify streets which form corresponding angles, alternate interior angles and alternate exterior angles.

E. Have groups of students develop lay outs of model communities on a very large sheet of paper. Each group should develop a geometric street grid with main streets running parallel or perpendicular to each other. Other cross streets may traverse main streets. Students might also want to include some circles, diagonals, parks, houses, cars.

2.7 Solve problems that relate geometric concepts to real world situations.

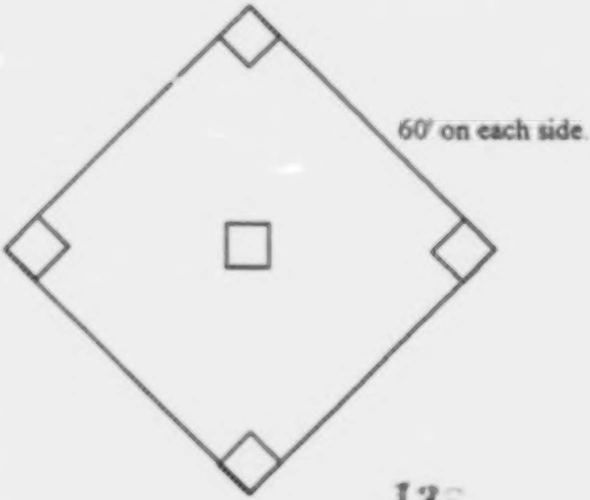
A. Have students work in small groups to design a playground using geometric solids. Draw an isometric three-dimensional representation.

Extension: Make other designs and drawings such as the school cafeteria, a classroom, or a community.

B. Props for science fiction programs often show space ships and space guns which are made from common household objects. For instance, space pistols are sometimes made from plastic spray bottles by cutting parts away and painting the rest. Give students this problem. If you were going to make a miniature space ship, what basic shapes would you need? What objects could you find to provide the shape and size needed?

C. Have students explore the design of a container using "Harold's Can Design." Have them work to design cans for a given volume with the minimum surface area and thus the minimum material costs. **See Activity Sheets 2.7C.**

D. Baseball and softball are popular sports in our country. The games are played on a diamond-shaped field. In softball, the distance from one base to another is 60 feet. As you can see from the diagram, the boundary of a softball diamond is square. When the maintenance crew lays out a softball diamond, they carefully measure the diagonals to make certain the diamond is really a square. How long should the diagonal be if the diamond is laid out properly?



E. To reinforce the concept of geometry in our environment, have students design a common sense house on a 10 x 10 grid using the information below. The area of the rooms should use the following amounts of floor space:

Living Room	20%
Kitchen	16%
Bathroom	10%
Bedrooms	18% each
Porch	8%

The remaining space is hallway and storage. How much space is hallway and storage? The rooms should be practical and liveable.

Discussion: Do rooms need to be quadrilaterals? What is the meaning of liveable room dimensions? Should the porches be on the exterior sides of the house? Is it a practical house if one has to go through certain rooms to get to other rooms? Should you have hallways that lead only to storage rooms?

Extension: Have students use a symbol of their choosing to indicate a doorway.

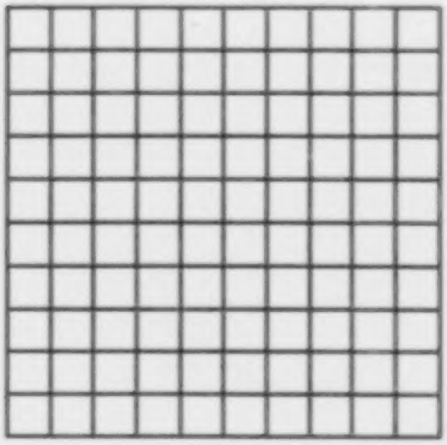
Change the grid to other dimensions.

Color code the rooms.

Write a description of your house.

Write a justification of room locations.

Change the floor area of each room.

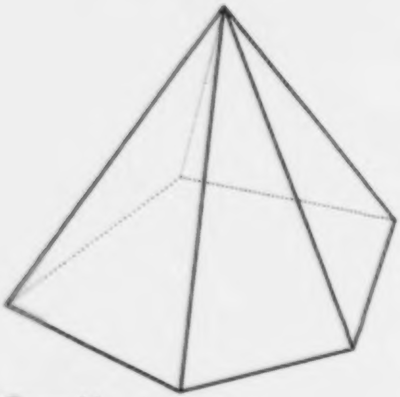


F. Take students on a geometry scavenger hunt to locate geometric shapes in the environment. **See Activity Sheet 2.7F.**

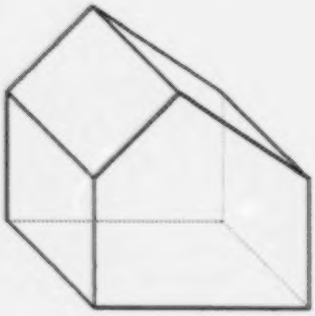
When you return from the hunt, discuss the following questions:
Why are wheels and manhole covers circular rather than square?
Why are most buildings rectangular rather than circular?
Why are road signs different shapes?
What geometric shape do you like best? Why? Sketch it.

G. Allow students to build a kite out of tetrahedra using the "High as a Kite" activity sheet. This will also enhance their ability to investigate properties of quadrilaterals. **See Activity Sheets 2.7G.**

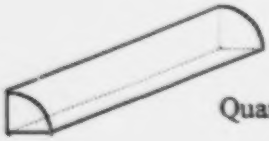
H. Have students keep a record of all pyramids they find as they travel around the city or state. They should either keep a journal of what the pyramid was used for or on what type of structure it was used. Photos would be good. The same idea could be used for other shapes such as spheres, prisms, or cylinders. Remind students that pyramids may have a base with more than four sides, prisms do not have to have all sides of the base equal, and sometimes half cylinders or quarter cylinders are used to decorate buildings.



Pyramid

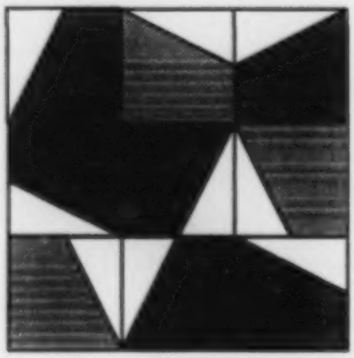


Prism



Quarter cylinder

H. Develop the students' awareness of geometry in everyday life and ability to relate their heritage to math by letting them work on a quilt project. You will need brown grocery bags taken apart at the seams and opened up flat. Have students use a right angle ruler to cut a square at least one foot on each side. They will need to divide it into a nine patch and draw those lines on the square. Using the dimensions of one of the smaller squares, let student make templates of the shapes they plan to use in each of the nine squares. For example, they may choose to draw a trapezoidal shape in one of the small squares. An individual template can be cut out for this shape. Then this shape can be rotated and flipped for use in other small squares. When the design is drawn, let the students color it with markers. To make the quilt, have students wad and unfold their square repeatedly until the square is very soft and looks stone-washed. Lace the big squares together with yarn or ribbon, then display in an appropriate place.



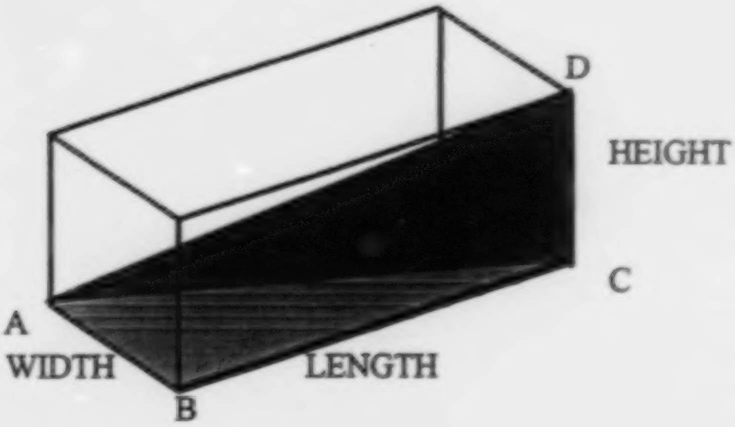
This represents one student's square which was designed using the shape below.

BLACKLINE MASTERS EIGHTH GRADE STRATEGIES

Corner to Corner
Activity Sheet 2.1A

Use the box that you brought from home and follow the steps in this activity to find the length of the longest object that will fit into your box. Work with your partner.

1. Measure and record the length, width, and height of your box.
2. Label the points on your box as shown in the diagram below.
3. Triangle ABC is a right triangle in the base of the box (prism). Find the length of AC, the base's diagonal.
4. Triangle ACD is a right triangle in the interior of the box. Use your answer to problem 1 to find the length of AD.
5. Compare the dimensions of your box to the dimensions of any box that had the same results that you and your partner found. What conclusions did you draw?



Extension: Have students find objects of length AD to see if they fit in their boxes. Have students find the longest object that will fit through the doorway of their classroom.

Television Screens
Activity Sheet 2.1B

Find whole number values of lengths and heights which would round to the diagonal screen size shown. Would these shapes be practical for TV screens?

Size of Screen	Length	Height	Practical?
19"			
19"			
19"			
19"			
20"			
20"			
20"			
20"			
26"			
26"			
26"			
26"			

Explain how you determined "Practical" or not.

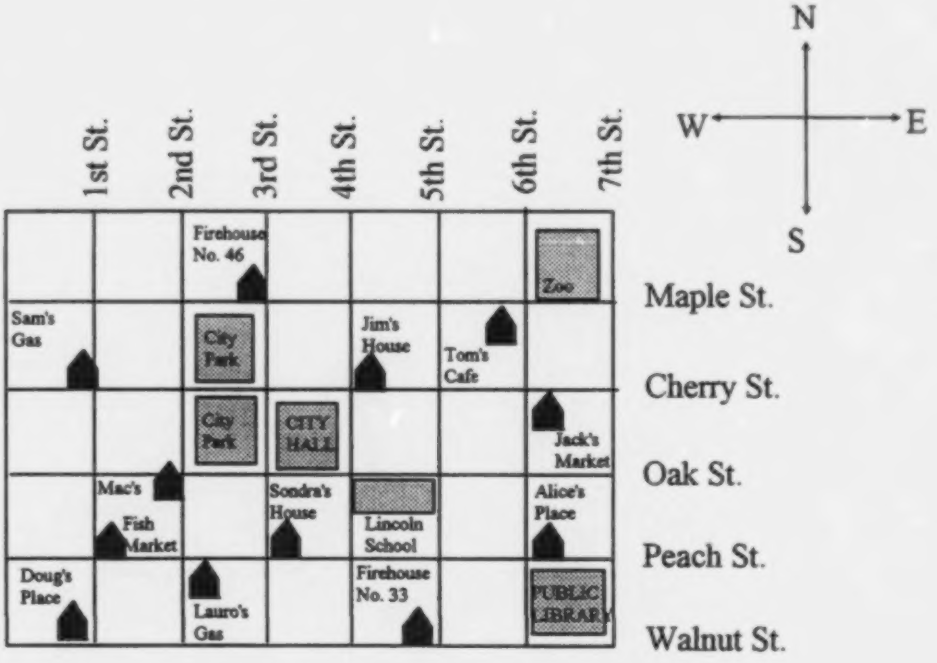
Taxi Cab Geometry and City Maps
Activity Sheet 2.1H

1. Joe is at the intersection of Walnut and Third Streets. If he takes a passenger to Alice's place, what is his road distance ? (The length of a city block is 1/8 mile.) What is the direct distance ?

2. Firehouse No. 46 responds to a fire at Jack's Market. What is the road distance? What is the direct distance?

3. Jim needs a ride from his house to the public library. Find the best route and the direct distance.

4. Make up a problem for someone else to solve. Why not throw in road construction and one-way roads or several stops along the way.


N
W
E
S
1st St.
2nd St.
3rd St.
4th St.
5th St.
6th St.
7th St.
Maple St.
Cherry St.
Oak St.
Peach St.
Walnut St.
Firehouse No. 46
Zoo
Sam's Gas
City Park
Jim's House
Tom's Cafe
City Park
CITY HALL
Jack's Market
Mac's
Fish Market
Sondra's House
Lincoln School
Alice's Place
Doug's Place
Lauro's Gas
Firehouse No. 33
PUBLIC LIBRARY

The figures on the map are located near corners where passengers might leave and arrive.

Extension: Doug needs to get to the zoo. He will walk only north and east. How many different routes are there?

Pythagorean Triples
Activity Sheet 2.1I

Pythagorean Triples are sets of whole numbers which will fit the Pythagorean theorem. One such combination is 3, 4, 5 because $3^2 + 4^2 = 5^2$. The following table gives a way to generate triples. Choose any two whole numbers m and n with m>n. The formulas at the top of each column generate one of the numbers in a Pythagorean triple set. Two examples are shown. Find as many Pythagorean triples as you can in which each of the numbers is less than 101.

m	n	a m^2-n^2	b $2mn$	c m^2+n^2	a^2+b^2	c^2
2	1	3	4	5	9 + 16	25
3	2	5	12	13	25+144	169

Right On
Activity Sheet 2.1J

This is a computer spreadsheet activity** in which students set up the spreadsheet with column headings and record data for two sides of a right triangle, and then use a formula to find the third side. Students will use critical thinking to determine the formula using transformations of the Pythagorean theorem. They will also have an opportunity to enhance their computer skills.

Materials
Set of data having dimensions of sides of a right triangle with one dimension missing, computer with spreedsheet software

Time
One class period

Description
Prepare handout with data
Turn on the computer
Open spreadsheet
Have students prepare spreadsheet as shown.

	Right On	
Leg A	**Leg B**	**Leg C**

Have students record their data leaving the spaces with missing data empty. Students will click in empty spaces and use the square root of one of the transformations of the Pythagorean theorem as the formula to find the missing data. The formulas will be:

=SQRT(a^2+b^2)
=SQRT(c^2-b^2)
or =SQRT(c^2-b^2)

Extensions: Allow students to test made up data to find out if the dimensions could make a right triangle by extending the spreadsheet with headings:

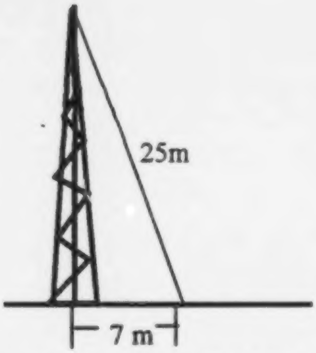
a^2 b^2 a^2+b^2 c^2 Right Yes/No

**This activity can also be done as a calculator activity with a chart to complete.

147

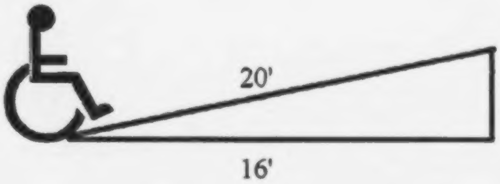
Pythagorean Theorem Applications
Activity Sheet 2.1K

1. A guy wire is 25 m long. It is attached to an anchor on the ground, 7 m from the base of the TV tower. How tall is the tower?

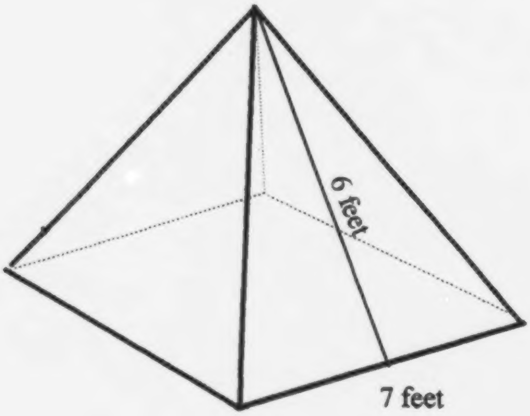


2. John left his campsite to go on a hike. He plans to keep in touch with his father by a walky-talky system which has a range of 5 miles. John walks 4.3 miles north and then 2.4 miles east. Will he be able to talk to his father?

3. A ramp is designed to help individuals in wheelchairs move from one level to another. What is the height of the ramp?

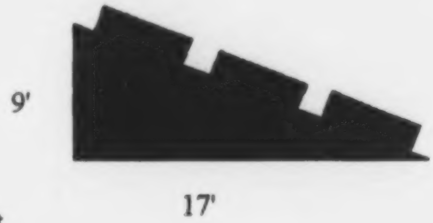


4. Mark is building a pyramid with a square base. Each side of the base is 7 feet long. The isosceles triangles that make the sides of the pyramid have an altitude of 6 feet. If Mark is 5'3" tall, can he stand up in the pyramid?

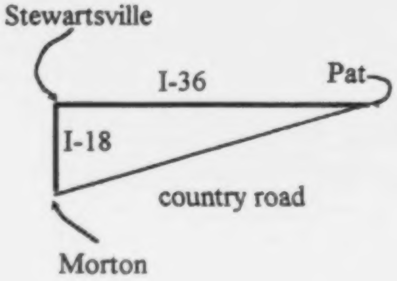


Pythagorean Theorem Applications
Activity Sheet 2.1K (continued)

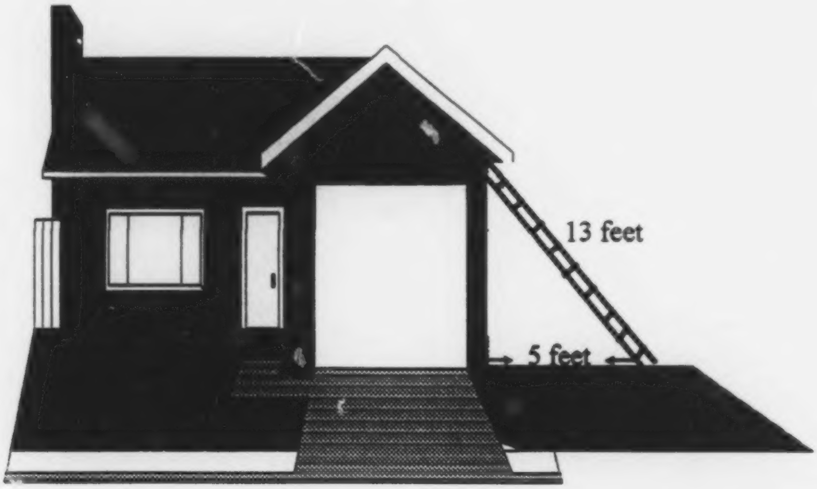
4. A conveyor belt moves boxes up this ramp. If the ramp has the lengths shown, how far do the boxes move along the belt?



5. Pat believes that Interstate 36 makes a right angle with Interstate 18 in Stewartsville. She is traveling along I-36 and plans to get on I-18 in Stewartsville and then travel down I-18 to Morton. She is now 38 miles from Stewartsville. Morton is another 15 miles from Stewartsville. Pat has found a country road going from her present location straight to Morton. If she gets off of the interstate, how many miles will she save?



6. How high up on the house does the ladder reach?

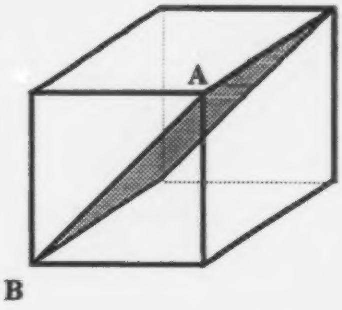


Fold, Drop and Pop Box
Activity Sheet 2.1M

Materials:
- scissors
- ruler
- rubber band
- pencil
- poster board or cardboard
- glue or paste
- coloring tools

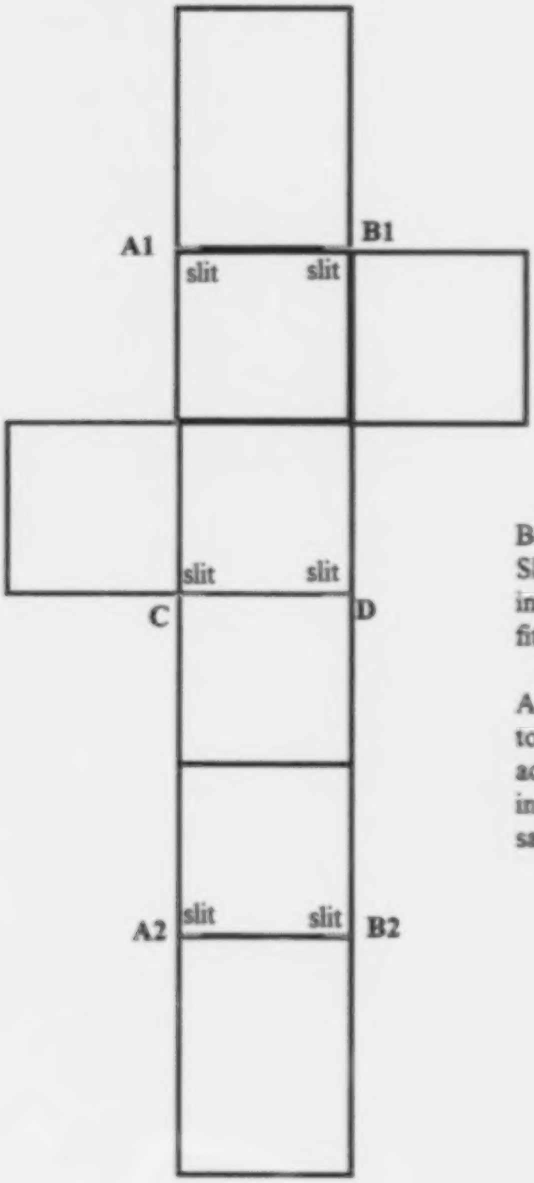
How:

1) Cut out the pattern provided.
2) Glue or paste the pattern to the cardboard.
3) Cut out the cardboard diagram. Be sure to cut the little slit indicated on pattern.
4) Score on fold lines.
5) Fold into cube shape -- the two longer rectangles at each end of the pattern form a diagonal "brace" in the center of the cube.
6) Problem: Fit the rubber band over the shape so that when the cube is flattened by pressing on points A and B (see diagram), the top and bottom are folded over the flat shape, the rubber band will cause the box to pop up.



Extension: Apply the Pythagorean theorem in order to create a similar pattern of a different size. Draw the pattern on a piece of cardboard using measurement skills and tools.

Fold, Drop and Pop Box
Activity Sheet 2.1M (continued)


A1
B1
slit
slit
slit
slit
C
D
A2
slit
slit
B2

Be sure to cut the slits. Slide the rubber band into the slits first, then fit along the diagonal.

A1 and A2 will be next to each other and will act as one slit when fitting in the rubber bands. The same goes for B1 and B2.

Mirror, Mirror on the Floor
Activity Sheet 2.2B

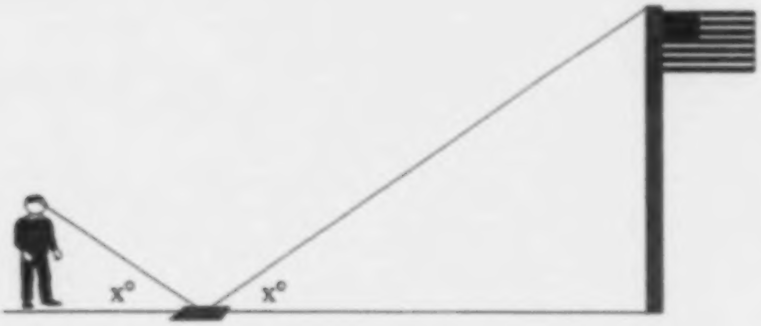
In this project you will find the height of a flagpole, or other designated object by using two similar triangles. You will need:

- a mirror, with its center marked some way
- a long measuring tape (50-100 ft.)
- pencil and calculator

Instructions:

You and your partner are going to do the experiment twice. When you are done, you can compare answers and discuss the differences, if any. Partner #1 will do steps 1 through 3; Partner #2 repeats steps 1 through 3.

1. Place the mirror on the ground a distance from the flagpole. Make sure you can see to the top of the flagpole in the mirror!
2. Now position yourself with the mirror between you and the flagpole so when you look at the mirror you see the top of the pole on the point marked on the mirror.
3. Have your partner make the following three measurements. Make the measurements to the nearest inch.



Measurement	**Partner #1**	**Partner #2**
Height of observer:	________	________
Distance from mirror center to observer:	________	________
Distance from mirror center to base of pole:	________	________

Mirror, Mirror, on the Floor
Activity Sheet 2.2B (continued)

Now you can compute the height of the flagpole by using a proportion for two similar triangles. Remember that triangles are similar when 1) all corresponding angles have the same measure and 2) when corresponding sides have the same ratio. Since the flagpole and the observer are perpendicular to the ground, what kind of angle is formed by each? ____

What is its measure? _______ What type of triangle is formed on each side of the mirror? ____________________

There is an important property in physics that says when light is reflected by an object, like a mirror, the angle of incidence is equal to the angle of reflection. Or, in other words, the light hits the mirror at the same angle as that formed when the light is reflected away from it. In geometry we say that the two triangles are similar because they have two pairs of corresponding angles congruent.

Since corresponding sides of similar triangles are proportional, we can write the following:

$$\frac{\text{person's height}}{\text{person's distance}} = \frac{\text{flagpole height}}{\text{flagpole distance}}$$

Now rewrite the proportion, but substitute your measurements, and use a calculator to compute the height of the flagpole for both partners.

Flagpole height by Partner #1: ______________

Flagpole height by Partner #2: ______________

Do your answers agree within a few inches? ________________

If not, why do you think they differ?

If there is a significant difference, go back and carefully redo steps 1 and 3 for both partners.

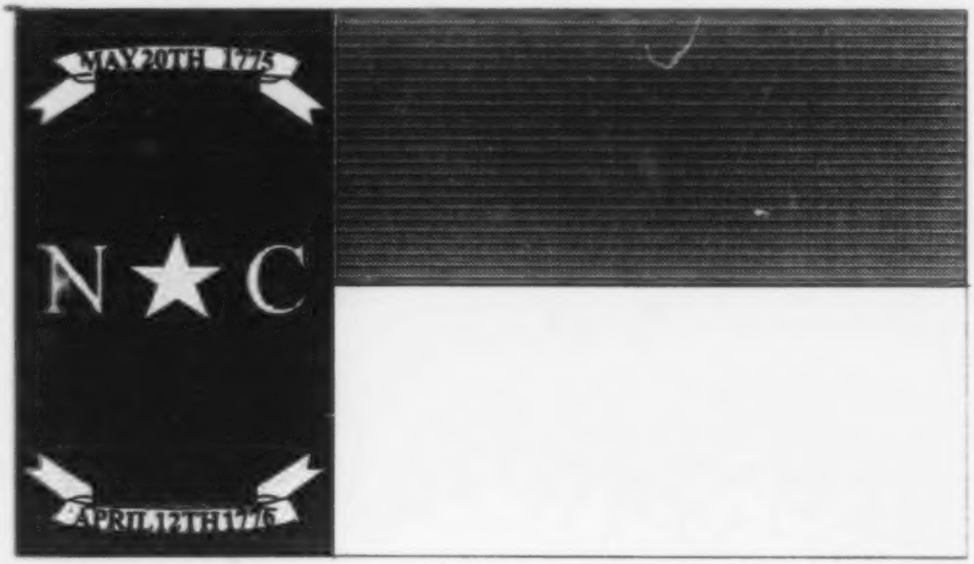
Carolina Capers
Activity Sheet 2.2C

Materials:
Worksheet of N.C. flag
Rulers
Calculators

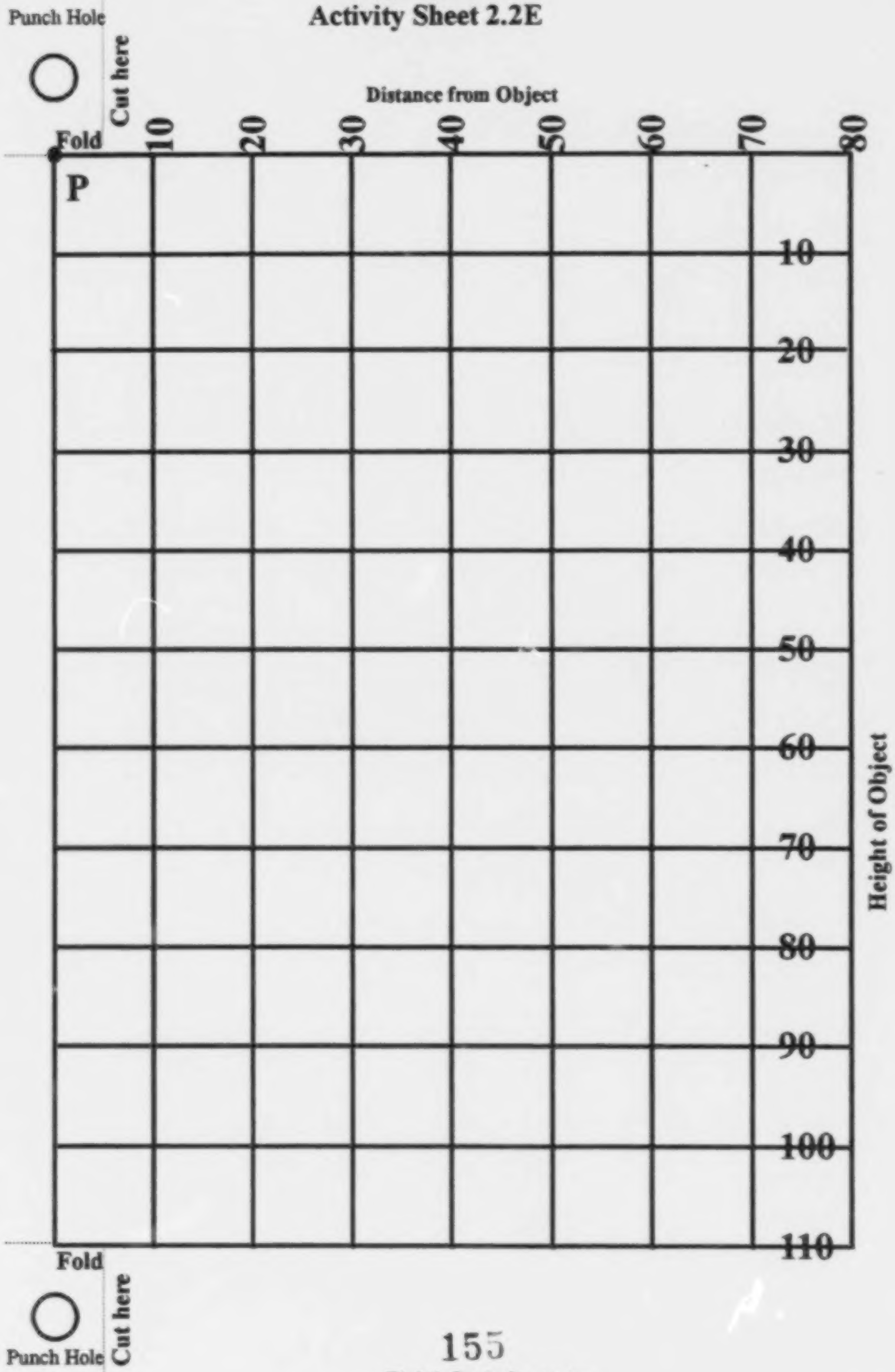
Problem:
Robert is going to make a larger scale of the North Carolina flag for his social studies project. He knows the total width of the flag will be 36 inches. Determine the width of the solid white section for Robert. Find the measures of other rectangles in the flag, size of the letters, star, etc. Students can also find area and perimeter of the original flag.

Plan:
Use ratio and proportions and the concept of similar figures to solve this problem.

Extension:
This activity may also be adapted to include historical landmarks such as the Outerbanks lighthouses, the Wright Brothers' monument, and the U.S.S. North Carolina.

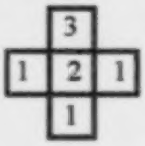


Hypsometer
Activity Sheet 2.2E

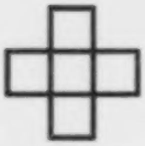

Punch Hole
Cut here
Distance from Object
10
20
30
40
50
60
70
80
Fold
P
10
20
30
40
50
60
70
80
90
100
110
Height of Object
Fold
Cut here
Punch Hole

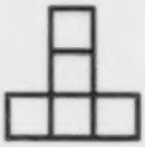
Draw Me in Three-D
Activity Sheet 2.3B

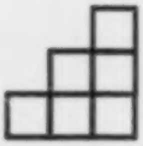
Use the plan below and cubic centimeter cubes to build a building. The numbers tell how high the centimeter cubes should be in that stack.



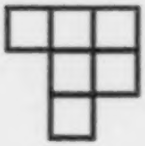
Make an orthographic drawing of your building.

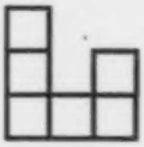

Top

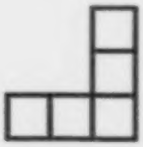

Front

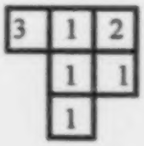

Side

Here is an orthographic drawing. Build the building.


Top


Front


Right Side



Building Plan

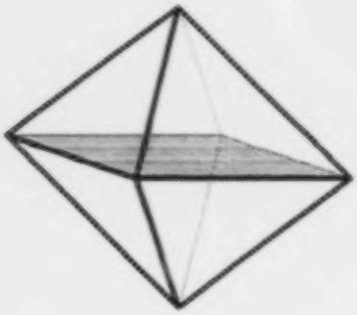
3-Dimensional Figures
Activity Sheet 2.3F

Materials: A number of things can be provided for building such as:
coffee stirrers to be fasthened together with pipe cleaners or
toothpicks to be fastened together with clay or marshmallows

1) Use your materials to build a cube.

2) A tetrahedron has 4 vertices and 6 edges. Build a tetrahedron. How many faces does it have?_________ Use your model to draw a tetrahedron.

3) Build an octahedron such as the one shown here.

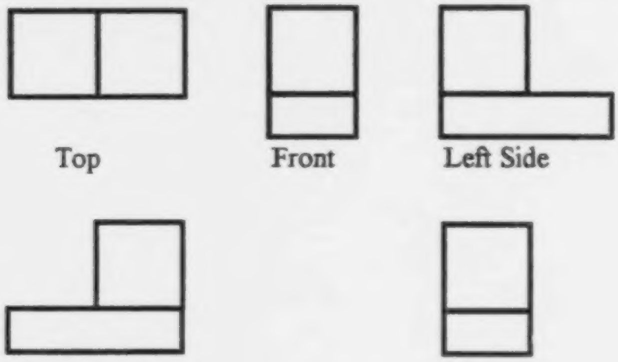


Extension: Build an icosahedron and/or a dodecahedron. Write a description of each of the figures you built. Make, draw, and describe other solid figures.

Build This
Activity Sheet 2.3H

Each problem is an orthographic drawing of a geoblock construction. Build each shape, name the shape, and write a description.

1.



Top

Front

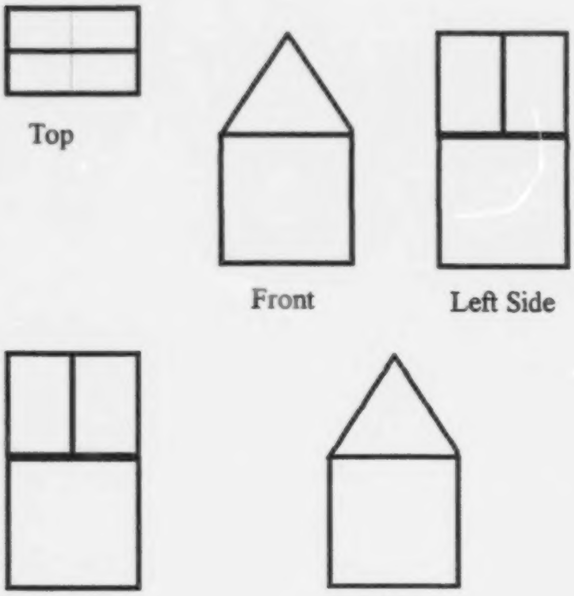
Left Side

Right Side

Back

2 Blocks

2.



Top

Front

Left Side

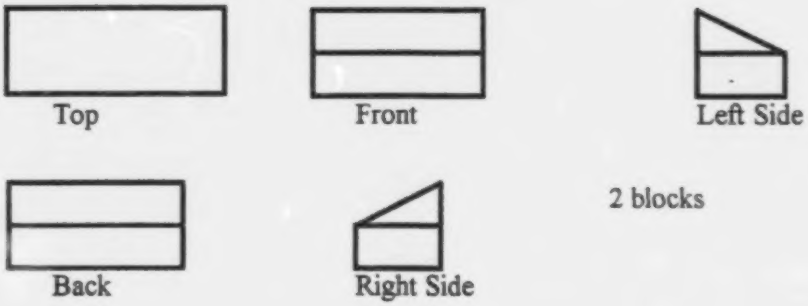
Right Side

Back

3 Blocks

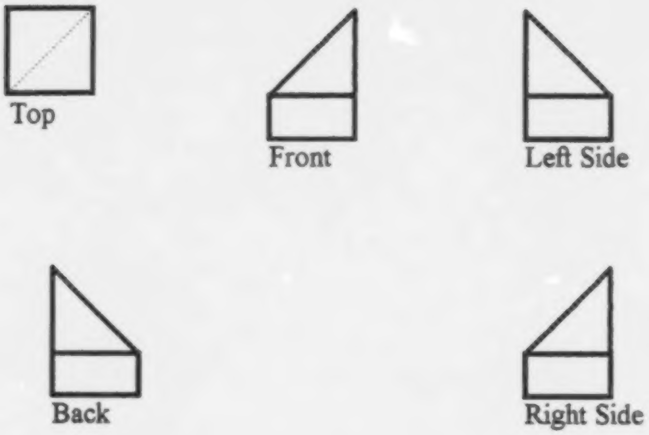
Build This
Activity Sheet 2.3H (continued)

3.


Top
Front
Left Side
Back
Right Side

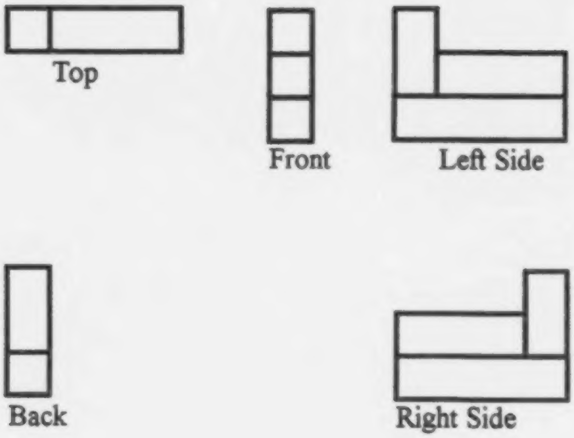
2 blocks

4.


Top
Front
Left Side
Back
Right Side

2 blocks

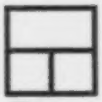
5.


Top
Front
Left Side
Back
Right Side

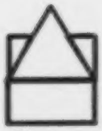
3 blocks

Build This
Activity Sheet 2.3H (continued)

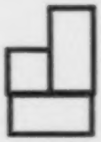
6.



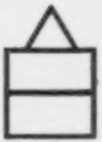
Top



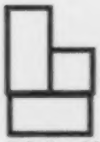
Front



Left Side



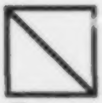
Back



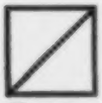
Right Side

3 blocks

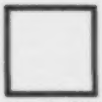
7.



Top

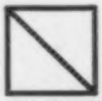


Front

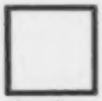


Left Side

3 blocks

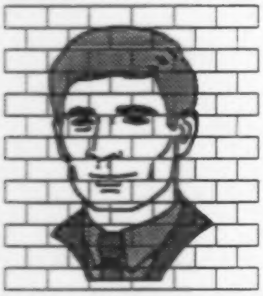


Right Side



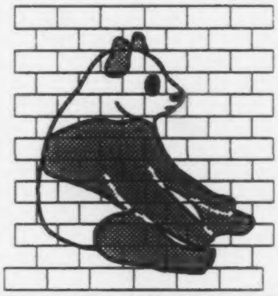
Back

Proportional Drawing
Activity Sheet 2.3L



Use the picture on the left and enlarge it by using the grid below.

Proportional Drawing
Activity Sheet 2.3L (alternate)



Use the picture on the left and enlarge it by using the grid below.

Table Top Transformations
Activity Sheet 2.4B

Materials:
- A large piece of 1" grid graph paper from a roll or 4 sheets of 1" grid graph paper taped together to form one large sheet
- Markers
- Rulers
- Scissors
- Index cards
- Charts TTT-1 and TTT-2
- Graph grids to record transformations

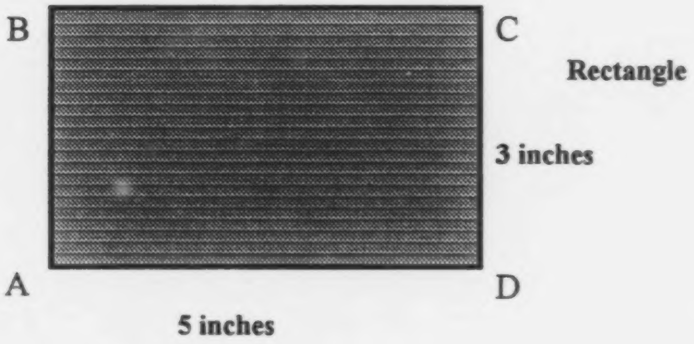
1) Write in your math notebook anything you know about slides, flips, and turns.
2) Compare notes with a partner and make additions.
3) Investigate and discuss correct vocabulary (translations, reflections, and rotations).
4) Use your large sheet of graph paper to make a coordinate plane. Show x- and y-axes. Label grid from -12 to 12.
5) Cut geometric shapes described below from 3 x 5 index cards.
6) Place the rectangle in the first quadrant with vertex A at (2,3) and B at (2,6).
7) Record the other coordinates in TTT-1 for the original position.
8) Complete the chart for the rectangle's translation, rotation, and reflection.
9) Remember to return to your original position after completing each transformation.
10) Repeat steps 7-10 for each polygon.
11) On separate graph paper, graph the original position, rotation, translation and reflection for each polygon.

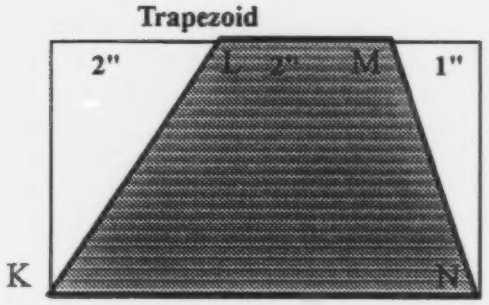
Extensions:
- Complete chart TTT-2
- Use different starting points, polygons, points of translation
- Rotate different degrees and directions
- Reflect across a line other than an axis, or rotate around a point other than the origin.
- Try a larger-than-life -size coordinate plane on the floor of your classroom. Use students to plot points.

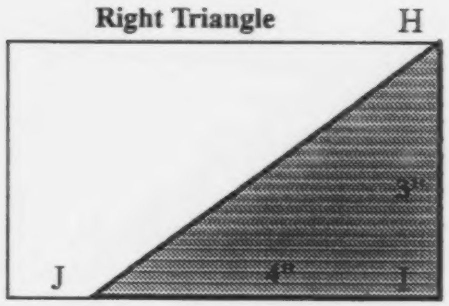
Concepts to Review:
- Placement and labeling of quadrants
- Geometric shapes, names and properties
- Writing ordered pairs
- Location of x- and y-axes
- Counterclockwise and clockwise

Table Top Transformations
Activity Sheet 2.4B (continued)

Shapes to use:


B
C
Rectangle
3 inches
A
D
5 inches


Trapezoid
2"
L
2"
M
1"
K
N


Right Triangle
H
3"
J
4"
I

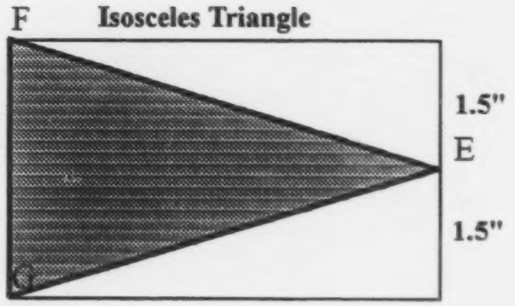

F
Isosceles Triangle
1.5"
E
1.5"
G

Table Top Transformations
Activity Sheet 2.4B (continued)
Sheet TTT-1

Give coordinates of original position, then the coordinates of the transformed shape. The rotation is 90° clockwise about the origin. The reflection is about the y-axis.

Shape Reflect	Original Position	Translate	Rotate	
Rectangle	A(2 , 3) B(2 , 6) C(,) D(,)	A(,) B(2 ,-4) C(,) D(,)	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)
Right Triangle	H(0 , 3) I (0 , 0) J (,)	H(,) I (2 , -4) J (,)	H(,) I (,) J (,)	H(,) I (,) J (,)
Isosceles Triangle	E(-2.5, 0) F (,) G (-1 ,-5)	E(,) F(,) G(-1 ,-3)	E(,) F(,) G(,)	E(,) F(,) G(,)
Trapezoid	K(,) L(6 ,-1) M(8 ,-1) N(,)	K(,) L(,) M(,) N(-1 , 1)	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)

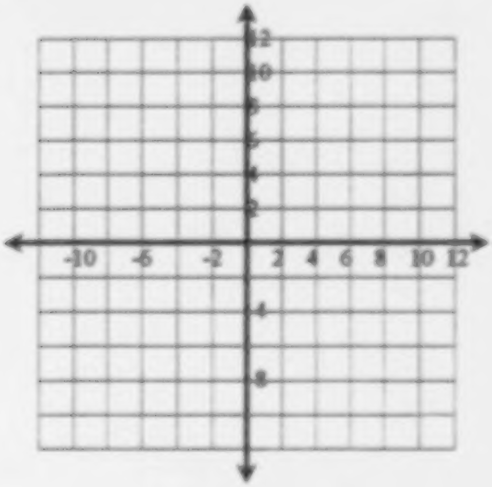
Table Top Transformations
Activity Sheet 2.4B (continued)
Sheet 2 (TTT-2)

Record coordinates of the original position of your choice, then the coordinates of the transformed shape. The rotation is 180° clockwise about the origin. The reflection is about the x-axis.

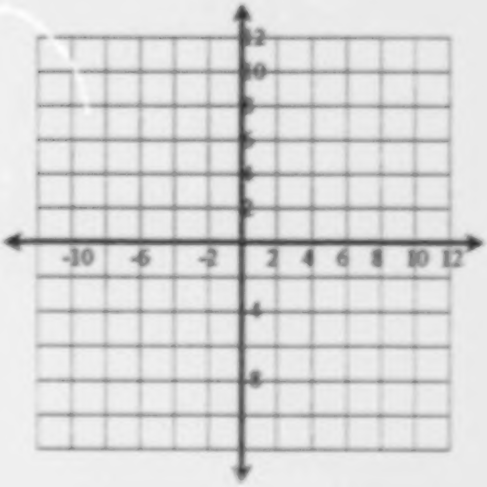
Shape	Original Position	Translate	Rotate	Reflect
Rectangle	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)	A(,) B(,) C(,) D(,)
Right Triangle	H(,) I (,) J (,)	H(,) I (,) J (,)	H(,) I (,) J (,)	H(,) I (,) J (,)
Isosceles Triangle	E(,) F(,) G(,)	E(,) F(,) G(,)	E(,) F(,) G(,)	E(,) F(,) G(,)
Trapezoid	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)	K(,) L(,) M(,) N(,)

Table Top Transformations
Activity Sheet 2.4B (continued)
Grid Recording Sheet

Polygon ____________


12
10
8
6
4
2
-10
-6
-2
2
4
6
8
10
12
-4
-8

Original Position


12
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2
-10
-6
-2
2
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6
8
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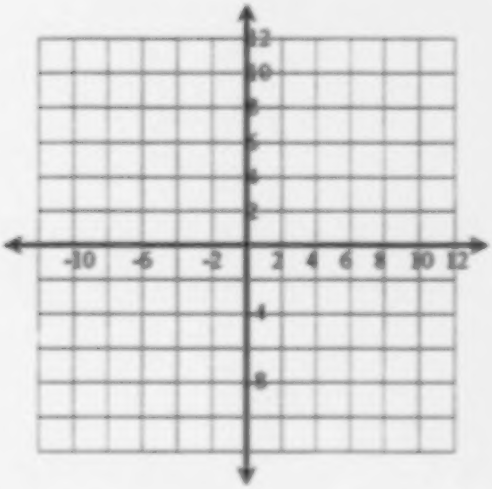
12
-4
-8

Translation


12
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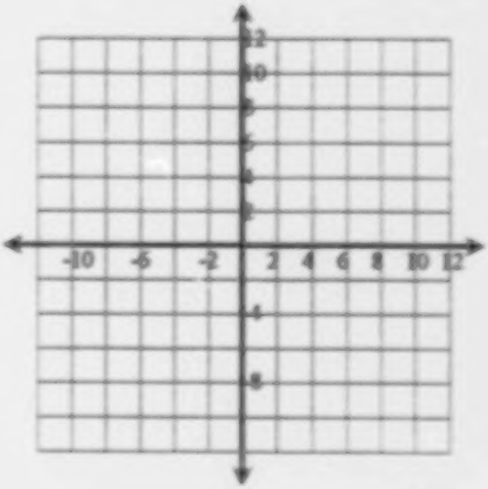
8
6
4
2
-10
-6
-2
2
4
6
8
10
12
-4
-8

Rotation


12
10
8
6
4
2
-10
-6
-2
2
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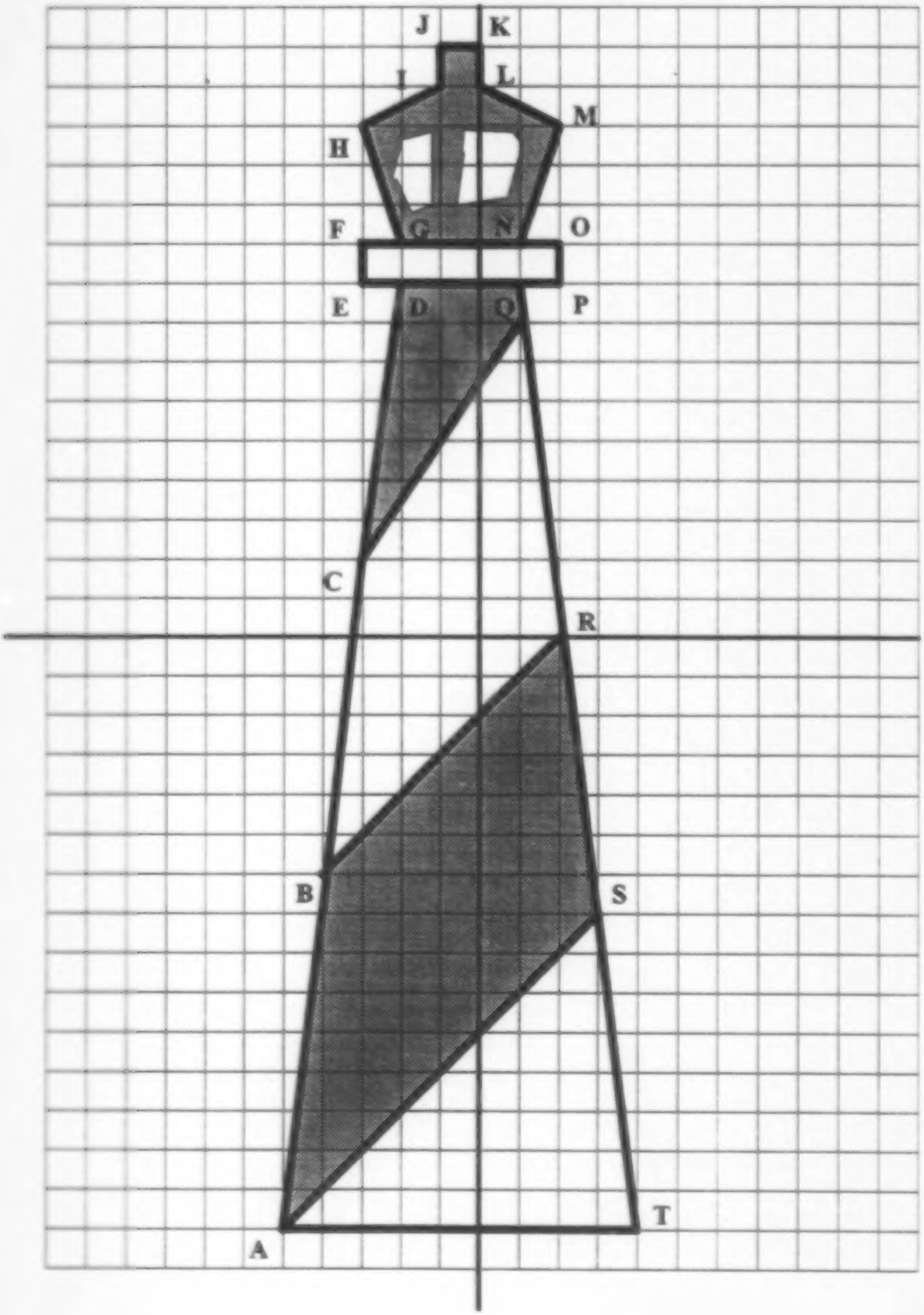
6
8
10
12
-4
-8

Reflection

Let Your Little Light Shine
Activity Sheet 2.4C


J
K
I
L
M
H
F
G
N
O
E
D
Q
P
C
R
B
S
A
T

Transformations in the Coordinate Plane
Activity Sheet 2.4J
Joseph Georgeson
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1991 Mathematics Institute

In the context of transformations, students will explore the effect addition and multiplication have on shapes drawn in the coordinate plane. Why is it, for example, that when something is added to the coordinates of a shape, it is not changed in size, only in position? Or how does multiplication of the coordinates of a shape affect the shape if the number is positive, negative, bigger than 1, less than 1 but bigger than 0, or equal to 1, or the result of adding numbers that are less than 0 to the coordinates?

The activities should be set up so that students are given the instructions for making a transformation rule. Their job is to work in cooperative groups and attempt to make conjectures about their rules, test the validity of their conjectures, and demonstrate to other groups what they have found. It is hoped that through the activities in this unit, students will come to a better understanding of arithmetic as it applies to this geometric model, and will have a better appreciation for the transformations that they see in the world around them.

Prerequisite:
·coordinate geometry (knowing how to locate points when the coordinates are given)
· area and perimeter on grid paper (informal knowledge is all that is required)
· other skills that students either have learned and forgotten, or never have learned at all
(Note: This is always the case in any class, so it is assumed that the teacher will provide appropriate instruction when necessary. For example, if students have never used variables, they will need some instruction. Some students, however, may have had exposure to variables, but in a different context. The teacher must determine what enabling instruction needs to be given.)

A discussion of the word "transformation" would be a good place to begin this unit. A general definition of transformation might be "change." Examples to be considered are slides, turns, flips, expansions, contractions, or combinations of these. The vocabulary that is used should be whatever students can understand at an intuitive level.

Students should be able to give many examples of transformations that they see around them. Have them give examples while a list is kept on the overhead. Keep that list during the time the unit is being done for reference. A few examples that might be mentioned (you should spend time informally discussing these examples and solicit from students many more examples):

· bricks on a wall, ceiling tiles, desks in a classroom, and blocks on a sidewalk can all be modeled by a translation or slide;

Transformations in the Coordinate Plane
Activity Sheet 2.4J (continued)

- propellers, certain letters of the alphabet, and Ferris wheels are examples of turns or rotations;

- people growing, the image of a shape on an overhead projector, or the image of a slide in a slide projector are all models of an expansion;

- melting ice, maps of the world, and model airplanes, are models of contractions; and

- mirrors, cars, people, and many geometric shapes model reflections or flips.

This list is not complete. Students should add to this list from their own experiences. Once they start, they will find many examples. This could be an ongoing activity from the beginning of the unit until the end of the year.

Activity 1:

On the first day bring magazines to class and have students identify pictures that represent each of the transformations. Make a bulletin board on which a collage of examples of the various transformations are displayed. Students could work in groups and put their pictures on the board in an organized fashion. One group could be responsible for "putting up" rotations or turns, while another group might be responsible for translations. Every group will find examples and funnel them to the appropriate group to decide how to display all of them.

End the class with discussion of their examples. Many questions could be asked, such as how they identified the transformation. What characteristics make a translation? How can you describe the rotation you found? What is special about a flip or reflection that tells you that it is one?

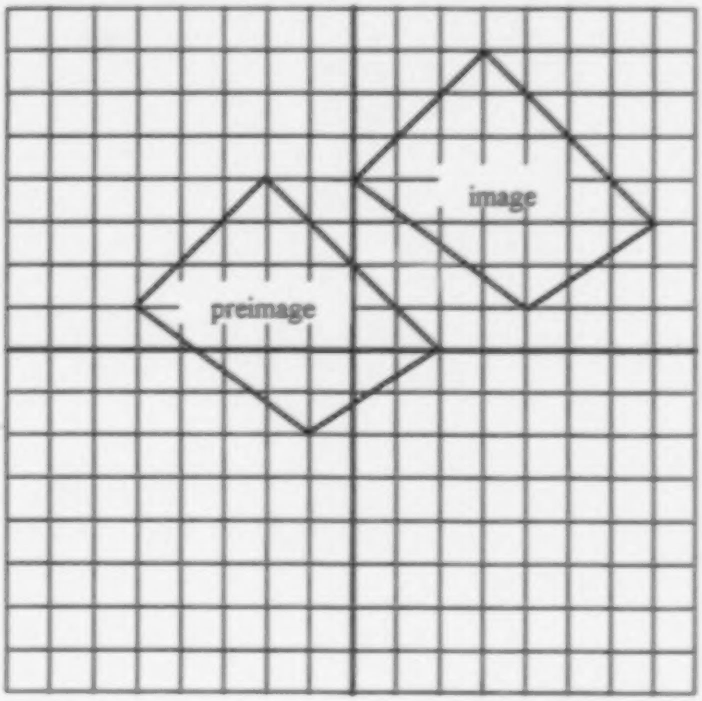
Activity 2:

The class will be exploring the transformations that they identified and displayed in Activity 1. However, they will be doing the exploration on graph paper using mathematical notation to describe each transformation. The same mathematics that works on graph paper is applicable to the real world models of transformations they have found. It is important to relate this mathematical model to the real world examples as often as possible. The following example introduces the notation that students will use during this unit. It gives students a working vocabulary to apply to each task so that they will understand what they are looking for when they work independently.

Transformations in the Coordinate Plane
Activity Sheet 2.4J (continued)

Vocabulary: preimage – the original shape
image – the transformed shape

Example 1: Demonstrate the following transformation:
$(x,y) \longrightarrow (x + 5, y + 3)$ means add 5 to the x-coordinate and 3 to the y-coordinate to each of the vertices in the preimage to build the image.



For example, the point (-1,-2) is transformed into the point (4,1) using the rule, "add 5 to the first coordinate, and add 3 to the second coordinate." The other vertices of the preimage are transformed in the same way.

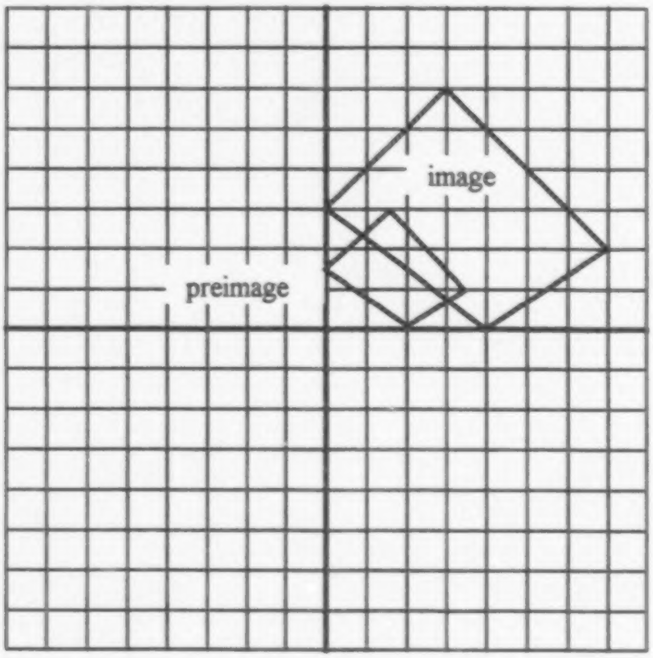
Note to teachers: Shapes can be restricted any way the teacher feels is necessary. For example, restricting shapes to the first quadrant might be necessary for some classes, although, in some cases it might be surprising to see what students can do and understand if the context is meaningful. The context of these transformations could serve as a good model to introduce in a meaningful way the idea of adding positive and negative numbers.

Transformations in the Coordinate Plane
Activity Sheet 2.4J (continued)

Questions:
In what ways are the image and preimage alike?
How are they different?
Did the same thing happen to every point, or just the corners?
Are the shapes similar? Congruent?
Are the lengths similar? Equal?
What do the numbers 5 and 3 have to do with anything?
What if 5 were subtracted instead of added?
Why didn't the shape get bigger -- we added something to the coordinates?
If corresponding vertices are connected, what shape results?
What questions can you (the student) ask?
Are there other things you noticed?
Are the shapes oriented the same way? (Is the top still the top?)
What if this transformation were done over and over again?
Many more are possible. Students could surely come up with a few.

Next, ask students to predict what will happen in the following transformation:
$[(x,y) \longrightarrow (x + 4) ,y + 4)]$
Why do you think that? How is this like the first one that was demonstrated? How could you verify your prediction?

Example 2: Demonstrate the following transformation:
$(x,y) \longrightarrow (2x,2y)$



Transformations in the Coordinate Plane
Activity Sheet 2.4J (continued)

Notes:
The image of (2,0) is (4,0), and so on. The coordinates in the image are found by following the rule, "double each coordinate." It is important to both write the rule using algebra (symbols) and say the rule using everyday language. This helps students see that one use of a "variable" is to describe in a convenient way a pattern or rule that has some use.

Other shapes could be graphed. It is important to pick shapes that fit on the graph paper you are using. Students could make up their own. Be careful (or aware) of shapes like squares or rectangles with lines of symmetry that make it difficult to see certain transformations.

Questions:
Is this an expansion or a contraction (did the image get bigger or smaller)?
What happened to the shape?
How are the image and preimage related (sides, angles, orientation)?
If corresponding vertices are connected, what shape results?
How is the area related? (This might be where a "guess" is acceptable, or simply the idea that the shape got bigger or smaller if the transformation is a contraction.)
What is the ratio of the length of the preimage to the length of the image?
How do the areas compare when expressed as a ratio?

Activity 3:

Give students adequate time, plenty of graph paper, and some rules to investigate. After doing the few you recommended, they should make up some rules on their own. Their job as they do this activity is to discover relationships between what is done to the coordinates and its effect on the image. The features they should look for are size (area and perimeter), orientation (top, bottom, left, and right), shape (similar, congruent, stretched, distorted), and others that students feel are important to name in describing what happened to the shape after it was transformed. Which of these features remained the same, and which were changed? How were they changed? The vocabulary of the students and the way in which they described the transformation should be accepted. Try to make them feel that the discoveries they are making are new and different and that you are surprised.

Transformations in the Coordinate Plane
Activity Sheet 2.4J (continued)

Suggested Transformations:

Students should investigate several classes of transformations:

a. Transformations that lead to expansions will be of the form:
$(x,y) \longrightarrow (ax,ay)$, where $a>1$.

b. Transformations that lead to contractions will be of the form:
$(x,y) \longrightarrow (ax,ay)$, where $0<a<1$.

c. Transformations that lead to reflections will be of the form:
$(x,y) \longrightarrow (ax,ay)$, where $a<0$. The size may change also.

d. Transformations that lead to translations or slides will be of the form:
$(x,y) \longrightarrow (x+a,y+b)$, where a and b are real numbers. Integers would be the easiest to consider.

e. Transformations of the form $(x,y) \longrightarrow (y,x)$ will lead to reflections, but the line of reflection will be the line $y = x$.

f. Other transformations that could be explored:
$(x,y) \longrightarrow (0,y)$
$(x,y) \longrightarrow (x,0)$
$(x,y) \longrightarrow (x,y)$
$(x,y) \longrightarrow (2,4)$

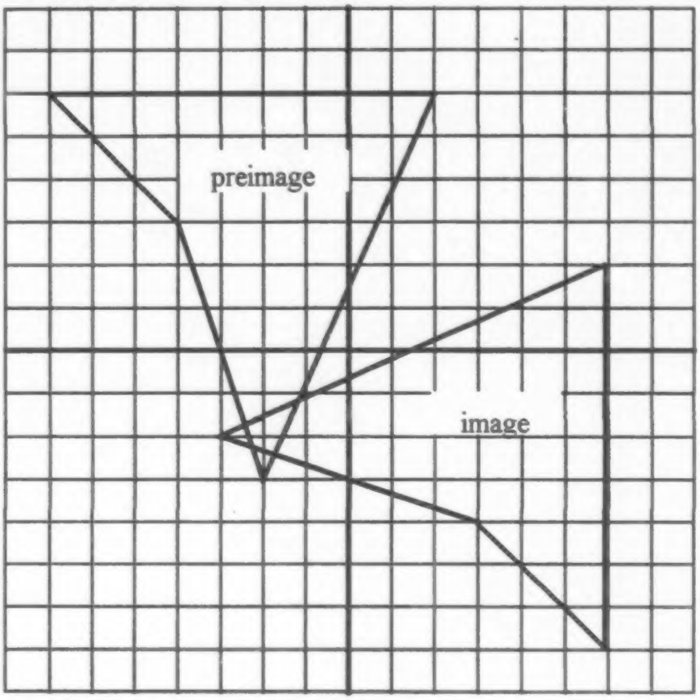
Summary Questions:

How does the image of a shape that is drawn change when the coordinates are changed according to some rule? What features of the shape are changed, and what features remain fixed? How could you describe the changes? How are the changes related to what was done to the coordinates? These are some questions that this unit attempts to answer. It is not important for all students to find all of the answers. It is, however, important for all students to explore these patterns and work with other students to better understand these ideas.

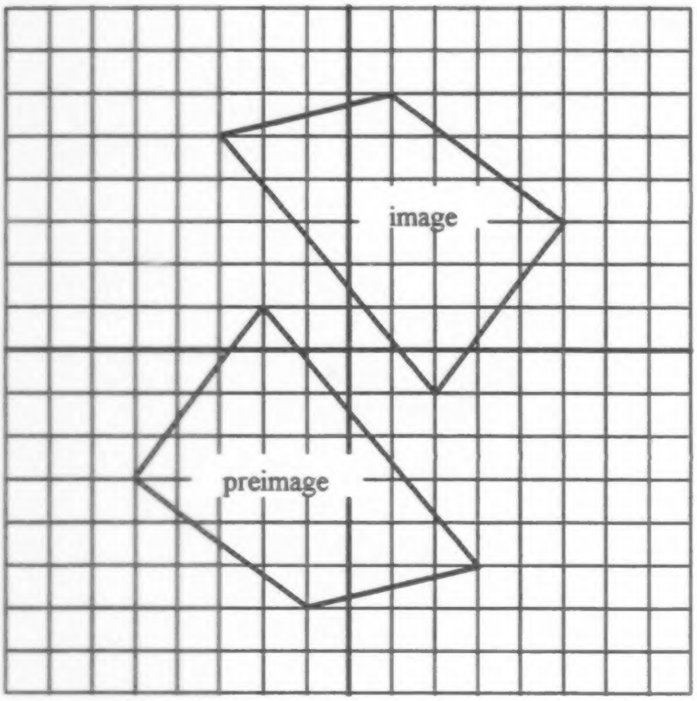
Transformations in the Coordinate Plane
Activity Sheet 2.4J (continued)

Some examples of what students should be able to do.

(x,y) → (y,x)


preimage
image

(x,y) → (-x,-y)



image
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Straw - Straw - Straw
Activity 2.5A

Materials: (Per student)
1/2 sheet of transparency film
1 overhead pen
scissors
1 plastic straw or coffee stirrer
Pipe cleaners to join the coffee stirrers

Instructions:

1) Instruct students to cut their straw into 3 sections. These sections may be the same size, but it would be better if they were all different sizes.
2) The students should arrange the straws to form a triangle on the transparency sheet. If some students cannot make a triangle out of their segments, discuss why not.
3) Instruct students to trace their triangle on the transparency.
4) Have students remove the straws and rearrange them on a sheet of paper in a different way. They are to try to create a different triangle.
5) Again, the students trace this, but on the paper, not on the transparency.
6) Now they compare this trace with the original trace on the transparency. Are they the same?
7) Have the students repeat this process several times until they are sure the results will always be the same.
8) Have the students discuss why this happens.
9) Introduce the SSS triangle congruency postulate.

Extensions: Along with the straw sections, assign one or two angle measurements to be used when the students construct straw triangles.

SAS -- The two straws are joined to make the given angle measurement, and the third side is drawn in. Repeat the process on paper. Compare the transparency triangle to the one on paper.

ASA -- Now the student selects one straw section and two different specific angle measurements. The angles of the given measurement are constructed at the ends of the straw segment. The angle sides are drawn and extended until they intersect to form a triangle. Again, do this on transparency and on paper and compare the results.

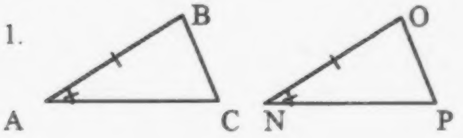
Note -- This activity can be completed in cooperative groups where the students exchange and compare triangles.

Triangle Congruency Practice
Activity Sheet 2.5C

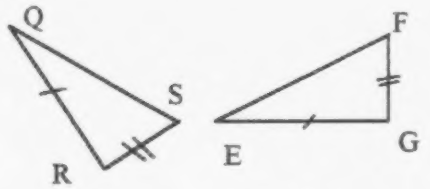
In the chart below identify one pair of congruent angles and/or sides which need to be added to qualify the triangle for each indicated congruency statement.

Note: Only 1 pair of congruent angles or sides should be identified for each congruency statement. If a triangle congruency is not possible with one additional pair of congruent parts, write NONE.

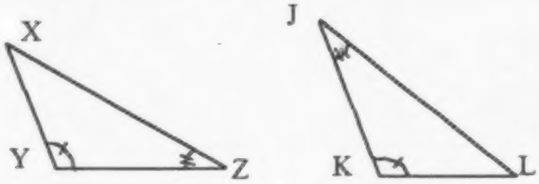
1.



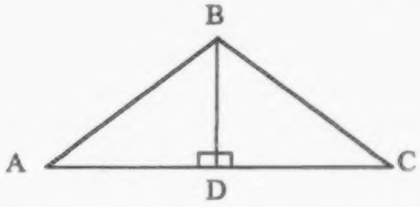
2.



3.



4.



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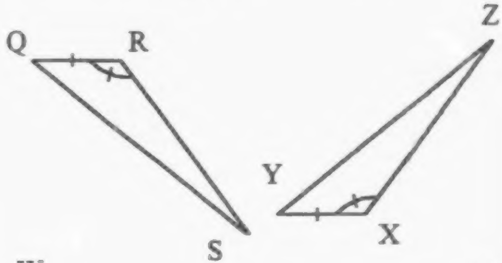
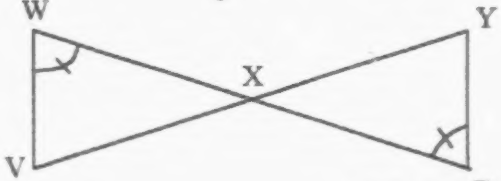


6.



SSS	ASA	SAS
1. NONE	<B ≅ <O	$\overline{AC} \cong \overline{NP}$
2.		
3.		
4.		
5.		
6.		

Ladders and Saws
Activity Sheet 2.6A

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This powerful tool for having students explore concepts related to parallel lines and polygons was developed by Dina van Hiele-Geldof in conjunction with her doctoral dissertation. "Ladders and Saws" provides a nice bridge between concrete manipulative activities and formal deduction. As an example of semi-deduction or informal logic, it is ideal for middle school students. It is also a good group activity.

Cut out any type of triangle from stiff paper, and color each angle a different color. (If each student has a different triangle, the conclusions reached seem even more powerful to them.) Draw a straight base line fairly near one edge of a blank sheet of unlined paper and line one edge of the triangle up along the base line. Trace one edge of the triangle away from the base line, and mark the point where the third vertex meets the base line. (See Figure 1.)

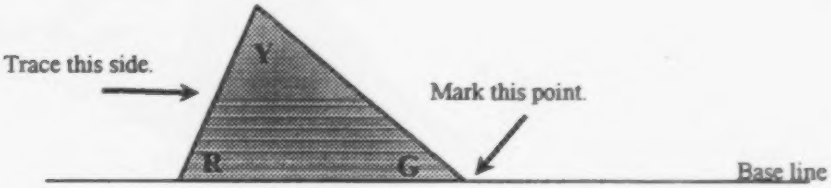


Figure 1

Then slide the triangle down the base line to the marked point and repeat -- trace the top edge and mark the lower vertex as shown in figure 2. Color in each angle as you make it.

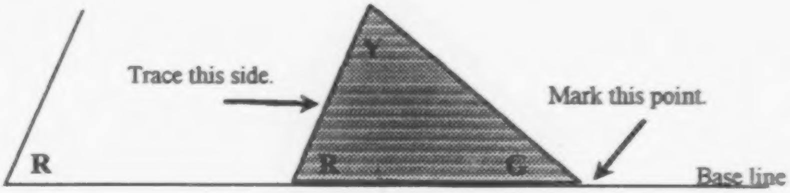


Figure 2

Continue this process all the way down the base line, creating a series of parallel lines, as shown in Figure 3. The resulting figure is called a "ladder."

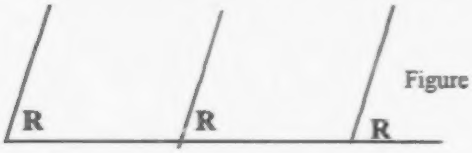


Figure 3

Ladders and Saws
Activity Sheet 2.6A (continued)

To create a "saw," line up the triangle along the base line and trace its other edges. Color in the angles as you go. (See figure 4).

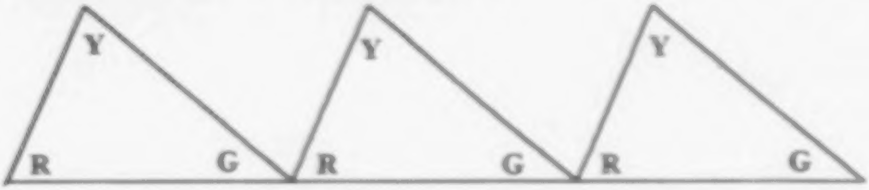


Figure 4

Now, rotate the triangle so that the vertex marked Y fits snugly against the base line, and the side marked YG fits against its tracing as shown in Figure 5. Then trace the top side.

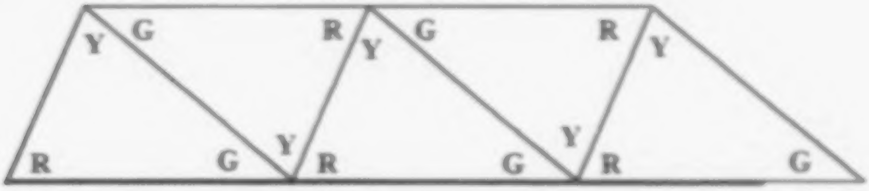


Figure 5

The "ladders" in Figure 3 illustrate corresponding angles formed by parallel lines cut by a transversal. The "saw" in Figure 4 shows alternate interior angles formed by parallel lines and parallel transversals. What conclusions can you make about each type of angle?

On a blank piece of paper, start with a base line, and make a row of ladders, coloring the angles as you go. Then turn your triangle around to complete the saws, again coloring in angles to match the original triangle. Repeat until the entire page is covered. Parts of two rows are shown in Figure 6.

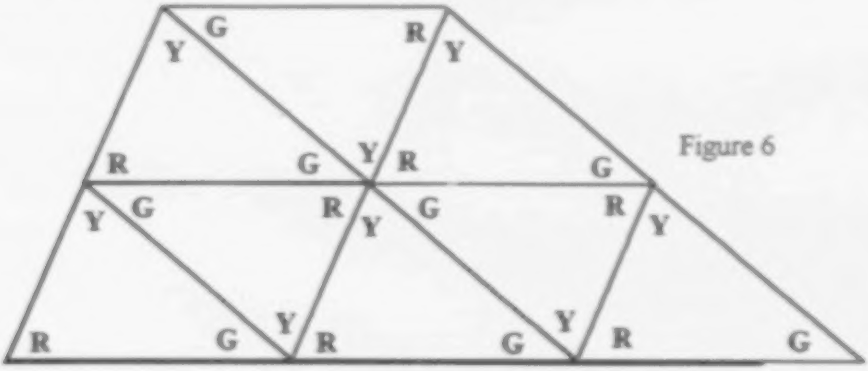


Figure 6

Ladders and Saws
Activity Sheet 2.6A (continued)

Once the page is covered with ladders and saws, look carefully at the figure and make as many conclusions as you can. You may want to outline or highlight certain parts of the figure, or cover up certain parts to make your conclusions stand out. How many conclusions can you make based on your diagram? Do your conclusions hold when you look at someone else's diagram? What would happen if you started with a special kind of triangle? Can you make any special conclusions about these figures?

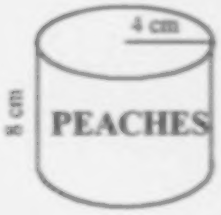
Some seventh grade students have come up with as many as 15 different conclusions. Can you top them?

Some Observations

1. The sum of the angles of any triangle equals 180°.
2. The sum of the angles of any quadrilateral is 360°.
3. The sum of the angles of any polygon is (n-2)180°.
4. The opposite angles of a parallelogram are always equal.
5. Angle size is preserved in similar figures.
6. Compare area of similar figures.
7. Circles are 360°.
8. Vertical angles are equal.
9. Alternate interior angles are equal.
10. Alternate exterior angles are equal.
11. Corresponding angles are equal.
12. Supplements of the same angle are equal.
13. An exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.
14. Triangles will tessellate.
15. Diagonals in a parallelogram bisect each other.
16. The diagonal of a parallelogram divides it into two congruent triangles.
17. Explore transformations, reflections, and rotations.

Harold's Can Design
Activity Sheet 2.7C

1. Calculate the volume of the can. (Round to the nearest whole number.)



2. Harold wants to design a can to hold 500 ml.

   a. Explain why $pr^2h = 500$.
   b. Explain why the surface area of the can, S, is given by $S = 2pr^2 + 2prh$.
   c. Harold decides to use a height of 10 cm.
      i. Work out the radius. (Do not round yet.)
      ii. Work out the surface area of the can. (Round to the nearest whole number.)
   d. Harold wants to find a design for the 500 ml can that will minimize the cost of the can (by minimizing the surface area). Complete the tables. (Round the surface area columns to the nearest tenth.)

h(cm)	r(cm)	$S(cm^2)$
12.5		
10.0		
7.5		
5.0		
2.5		

   e. After analyzing the chart in 2d, explain which design Harold will use.

Harold's Can Design
Activity Sheet 2.7C (continued)

f. Draw the container.

Extension: Measure the radii and height of various cans that students have brought in. Is their design optimal? If not, why not?

Reference: Mathematical Investigations by Randall Souviney et. al. pp. 19-23. (Dale Seymour Publications).

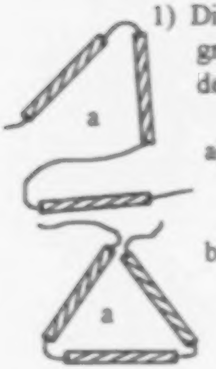
Geometry Scavenger Hunt
Activity Sheet 2.7F

OBJECT	WHERE	USE?	DRAW IT
Line segment			
Cylinder			
Square			
Sphere			
Parallel lines			
Perpendicular lines			
Circle			
Tessellation			
Prism			
Pyramid			
Octagon			
Trapezoid			
Similar figures			
Cone			
Non-parallel lines			
Vertical angles			
Right angle			
Triangle			
Semicircle			

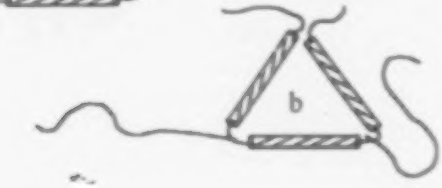
High as a Kite
Activity Sheet 2.7G

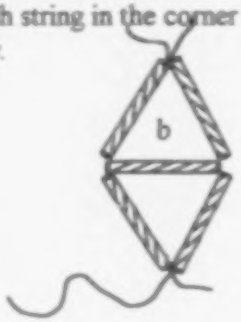
1) Divide students into groups of four. Have each member of the group construct a tetrahedron from straws and kite string as described below.

   a) Place 3 straws on a piece of string which is 1 meter in length. Form a triangle with the straws, but do not tie the string yet.

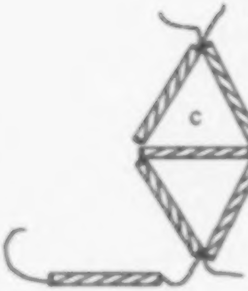


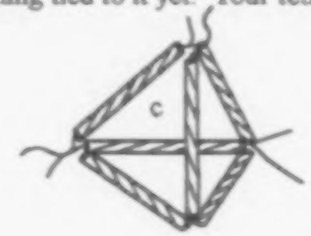
   b) Tie a 35 cm long string onto each corner of the triangle. Place a straw onto each string in the corner and tie the 2 strings together tightly.



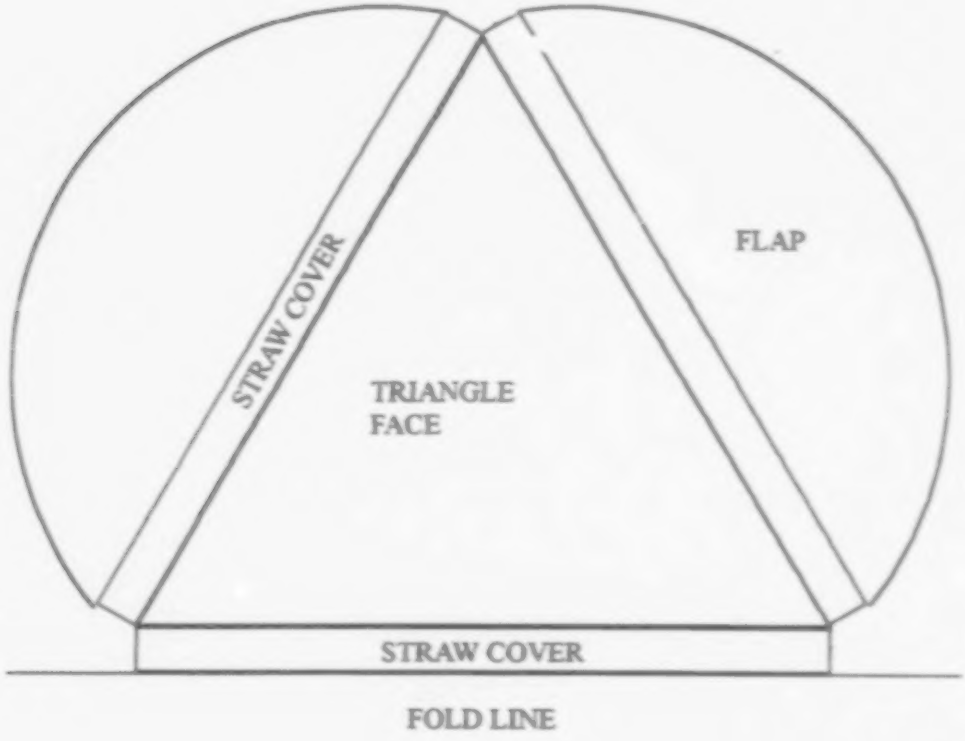


   c) Place another straw onto the loose ends of the string. Tie the end of this string onto the vertex of the original triangle that has nothing tied to it yet. Your tetrahedron should be completed.





High as a Kite
Activity Sheet 2.7G (continued)



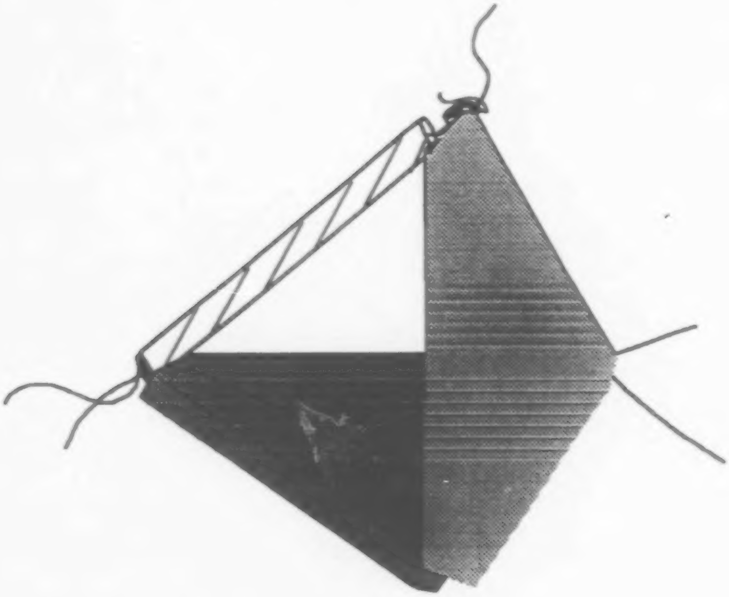
Enlarge the above pattern on a copy machine to twice this size by setting the enlarge factor to 200%. (You could also use an overhead or an opaque projector. The straw cover should be the length of your straw, approximately 8 inches.)

Place the pattern on a folded sheet of paper with the pattern's fold line on the fold of the paper. Cut out the pattern shape, but not the fold. The resulting shape should cover two sides of a tetrahedron.

High as a Kite
Activity Sheet 2.7G (continued)

2) Construct the "cover" for the tetrahedron by following the instructions on the pattern and then apply the cover to the tetrahedron.

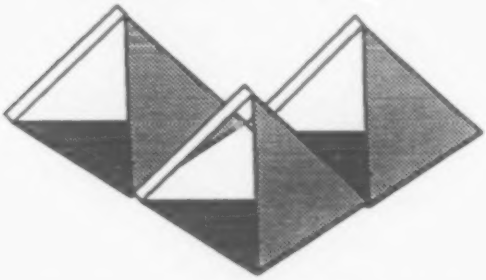
    a) Place a small amount of glue along the fold in the center of the cover, and lay one straws of the tetrahedron along the fold.
    b) Glue one side of the cover over one of the triangular faces of the tetrahedron. Fold the flaps over the straws to the inside of the figure.
    c) Tilt the tetrahedron and repeat the process for the other side. Each tetrahedron will have two sides covered with paper.



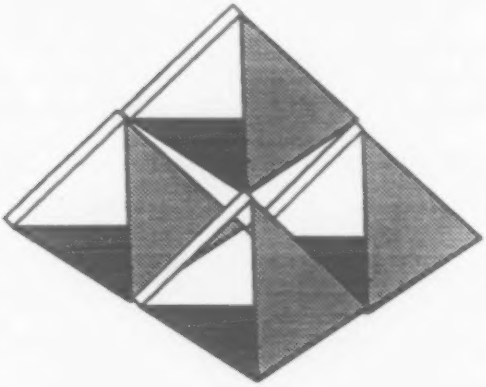
High as a Kite
Activity Sheet 2.7G (continued)

3) Put the four tetrahedra together to form the kite.

   a) Place three of the tetrahedra together in the shape of a triangle with all the covers facing the same way.



   b) Use string to tie the three tetrahedra together tightly at the corners where they touch.
   c) Place the fourth tetrahedron above the other three, facing the same direction, and tie it into place.



4) Attach the string to one of the tetrahedra on the lower level and "go fly a kite."
5) You may use this activity to incorporate scientific principles into your lesson.

GEOMETRIC THINKING DEVELOPMENT

The work of two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof, has had an impact on geometry instruction in the United States. Dina van Hiele-Geldof and Pierre van Hiele were high school teachers in the Netherlands who were disturbed by the way their students learned geometry. They believed that the geometry they were teaching in grades 7-9 involved thinking at a relatively high level, and that their students had not had sufficient experience in thinking at prerequisite lower levels. They were interested in finding ways to develop the student's level of geometric insight. Thus their research focused on levels of thinking in geometry, and the role of instruction in helping students move from one level to the next. They devised a structure for helping students develop insight into reasoning in geometry. The most prominent feature of the van Hiele model is the existence of five different levels of geometric thought within which one learns, talks about, and works with geometric ideas. Movement through the levels is sequential: when a person has achieved and solidified an understanding of geometric ideas at level n, he or she begins to construct understanding at the (n + 1) level.

The van Hieles stated that students will not learn concepts that are presented above their current level of thinking. When taught and asked to work and reason at a level they have not yet reached, students will perform the tasks by rote with no real lasting understanding. Furthermore, students cannot jump levels, i.e., move from level 1 to level 3 merely to catch up to the instruction.

For middle grade students, attention must be focused on levels 0, 1, and 2 of the van Hiele model which are discussed later. If students are having trouble learning geometry, one might hypothesize according to this model that they are being taught at a higher level than they have attained.

LEVELS OF GEOMETRIC THINKING

Level Zero

- Students identify shapes visually, without reasoning about their properties.
  Emphasis is on shape and from experiences.
  Properties of figures are included, but they are only explored informally.

- Students recognize and name figures in a global manner.
  They know a shape is a square because it looks like a square.
  Identify parts of a figure but do not analyze a figure in terms of its components.
  Identify instances of a shape by its appearance as a whole.

- The student identifies, names, compares and operates on geometric figures (e.g., triangles, angles, intersecting or parallel lines) according to their appearance.
  They can identify a shape and reproduce it.

- Students are influenced by nonrelevant attributes of a figure such as its orientation. They say such words as triangle and square but do not recognize properties of these figures.

- Students name or label shapes and other geometric configurations and use standard and/or nonstandard names and labels appropriately. They construct, draw, or copy a shape.

- Students solve routine problems by operating on shapes rather than by using properties which apply in general.

Level One

- Students see properties and relationships of figures as being interconnected and begin to exhibit deductive reasoning at an informal level.

- Students begin to look at specific characteristics of a figure. They identify those properties used to characterize one class of figures to another class of figures and compares classes of figures according to their properties. They recognize that certain characteristics set a shape apart from other shapes. For example, they know that a figure is a triangle because it is a closed figure and it is made with three line segments.

- Students analyze figures in terms of their components and relationships among components and discover properties/rules of class shapes empirically (e.g., by folding, measuring, using a grid or diagram). They may know that all sides of a square are congruent and that the diagonals of a rhombus are perpendicular bisectors of each other.

- Students identify and test relationships among components of figures (e.g., congruence of opposite sides of a parallelogram; congruence of angles in a tiling pattern).
  Students recall and use appropriate vocabulary in components and relationships (e.g., opposite sides, corresponding angles are congruent, diagonals bisect each other).

- Students interpret verbal or symbolic statements of rules and apply them. They discover proprieties of specific figures empirically and generalize properties for that class of figures. They interpret and use verbal descriptions of a figure in terms of its properties and use this description to draw/construct the figure.

- Students discover properties of an unfamiliar class of figures. They formulate and use generalizations about properties of figures (guided by teacher/material or spontaneously their own) and use related language (e.g., all, every, none) but do not explain how certain properties of a figure are interrelated. They do not formulate and use formal definitions, etc.

Level Two

- Students look at properties within figures and recognize the value of a definition. Students find that an equilateral triangle is isosceles because an isosceles triangle has two congruent sides and an equilateral triangle has three sides that are congruent so it must have at least two sides congruent.

- Students logically interrelate previously discovered properties/rules by giving or following informal arguments. They may relate figures and their properties, but do not organize sequences of statements to justify their observations. They may know that all squares are rhombuses but may not be able to state why in an organized way.

- Students formulate and use a definition for a class of figures. They identify different sets of properties that characterize a class of figures and test that these are sufficient. They give informal arguments using diagrams, cutout shapes that are folded, and other materials.

- Students recognize the role of deductive argument and approach problems in a deductive manner but do not grasp the meaning of deduction in an axiomatic sense (e.g., does not see the need for definitions and basic assumptions). Having drawn a conclusion from given information, they justify the conclusion using logical relationships. Students give more than one explanation to prove something and justify these explanations by using family trees.

- Students discover new properties by deduction. They give informal deductive arguments and can supply parts of the argument. They also give a summary or variation of a deductive argument.

Level Three

- This is the level that is used most often in high school geometry courses.
- Students can construct proofs not just recall those illustrated.
- Students at this level can reason deductively within the mathematical system to justify their observations.
- Students use models and work with an axiomatic system, that includes undefined terms, definitions, postulates, and theorems to support their arguments.
- Students prove theorems deductively and establishes interrelationships among networks of theorems. They compare and contrast different proofs of the same theorem by examining related statements, and the effects of changing the sequence of definitions, postulates, and previous theorems on the logic.
- Students make formal deduction arguments based on an axiomatic system but do not compare axiomatic systems.

Level Four

- Students at this level compare different axiom systems such as Euclidean and non-Euclidean geometries with a high degree of rigor without the use of concrete models.
- Students perceive geometry as an abstract system of logic and are able to create an axiomatic system for a geometry. They establish the consistency of a set of axioms and the independence of an axiom within a geometry system.
- Students spontaneously explore how changes in axioms affect the resulting geometry.
- Students invent generalized methods for solving classes of problems.
- Students search for the broadest context in which a mathematical theorem or principle will apply.
- Students do in-depth study of logic to develop new insights and approaches to logical inference.

PIERRE VAN HIELE'S PHASES OF LEARNING

According to Pierre van Hiele in his work which extended from 1959 to 1984 progress from one level of learning to the next involves five phases: information, guided orientation, explicitation, free orientation, and integration. The phases, which lead to a higher level of thought are described as follows.

Information: Here the student gets acquainted with the working domain, and examines examples and non-examples. Some introductory material and /or problem situation is presented. In this phase teachers should use many physical models that can be manipulated by the students. These activities involve repeated sorting, identifying, and describing of various shapes. Opportunities to build, make, draw, put together, and take apart shapes should be provided. Students should use objects in activities and make observations. Teachers should encourage conversation among students and guide students to examine how things are alike and how they are different.

Guided orientation: Here the student does assignments in a problem-solving context that involves different relationships within the network being developed. Tasks are sequenced by the teacher to gradually present concepts and uncover connections so that students learn to examine specific attributes of the object being studied. Teachers should include models that allow students to explore the various properties. Models provide students a concrete medium from which they define, measure and observe specific attributes. Students also should classify geometric figures based on the properties of shape as well as names of shapes.

Explicitation, Discussion and Reflection. The student becomes conscious of relationships, tries to express them in words, and learns technical language which accompanies the subject matter (e.g., expresses ideas about properties of figures). This is a time for both teacher and students to pull together the main ideas from the explorations. Customary mathematical terms are employed by the teacher. At this point the network of connections between mathematical ideas is being enhanced. The teacher encourages students to share their perceptions of structures, thus developing their language skills. The teacher's role is to help students use accurate and appropriate language. Activities should use models with a focus on defining properties. Students should make property lists and discuss which properties are necessary and which are sufficient conditions for a specific shape or concept. Activities should include language of an informal deductive nature: (e.g., all, some, none, if then, what if, and so forth). Activities should enable students to use models and drawings to help their thinking processes.

Free Orientation: By doing more complex tasks, students learn to find their own way in the network of relationships. This would include knowing properties of a specific geometric shape such as a rhombus and investigating whether these properties apply to a new shape such as a kite. Further tasks and activities help solidify the ideas that were newly formed in the guided orientation stage. Although the teacher supplies the materials for a task, students are left to work with the materials in their own way to complete the task.

Integration: Students summarize all that they have learned about the objects by reflecting on their actions and observations to advance their understanding of these relationships. Teachers provide global insights to help students make connections explicit, but do not introduce new ideas. This phase of instruction is designed for review and summary. Students condense the content into a whole to integrate what has been explored and discussed.

Appendix B

Glossary of Geometric Terms

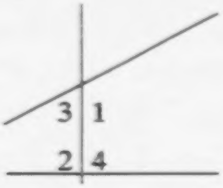
acute angle. An angle with a measure between 0° and 90°.

acute triangle. A triangle with three acute angles.

adjacent angles. Two coplanar angles with a common side, a common vertex and no common interior points.

adjacent sides of a polygon. Two sides of a polygon with a common endpoint.

alternate interior angles. A pair of nonadjacent angles on opposite sides of a transversal that lie between the two lines crossed by the transversal, such as angles 1 and 2 in the diagram below.

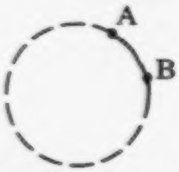


altitude (of a triangle). A segment from any vertex of the triangle perpendicular to the line on which the opposite side lies.

angle. A figure formed by two rays that have the same endpoint. The rays are called sides and their common endpoint is called the vertex. An angle is named using the ∠ symbol followed by three letters: any point (other than the vertex) on the first ray, the name of the vertex, and the name of a third point (other than the vertex) on the second ray. Thus the notation $\angle BAC$ represents the angle formed by rays *AB* and *AC*.

angle bisector. The ray that divides an angle into two congruent, adjacent angles.

arc of a circle. A part of a circle determined by two points on the circle.



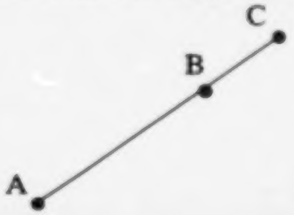
area. The measure of the interior of a closed figure recorded in square units.

axiom. A statement accepted without proof; an assumed rule.

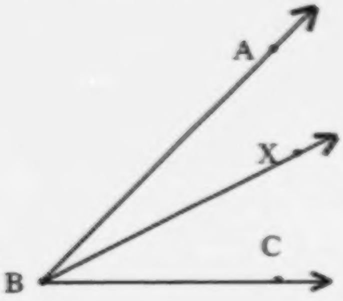
base of a triangle. Any side of a triangle. (In a non-equilateral, isosceles triangle, the unequal side)

base of a trapezoid. Either of the two parallel sides of a trapezoid.

betweenness of points. Point *B* is *between* points *A* and *C* if and only if *A, B,* and *C* are distinct and collinear and the distance from A to B plus the distance from B to C equals the distance from A to C. *(AB + BC = AC)*. See also *collinear points.*



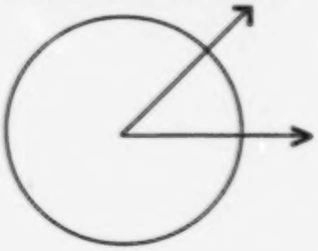
betweenness of rays. Ray *BX* is between rays *BA* and *BC* if and only if point X is in the interior of $\angle ABC$. Therefore, $\angle ABC = \angle ABX + \angle XBC$. See also *ray.*



bisector of a segment. A line, plane, segment, ray, or point that contains the midpoint of the segment and divides the segment into two equal parts. See also *midpoint of a segment.*

center of a circle. The point in a plane equidistant from each point on a circle. See also *circle.*

central angle of a circle. An angle whose vertex is the center of the circle. See also *vertex.*



centroid of a triangle. The point at which the medians of a triangle intersect. The point is the triangle's "center of gravity." See also *median of a triangle.*

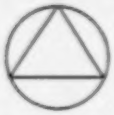
chord. A line segment whose endpoints lie on a circle.

circle. The set of points in a plane at a constant distance from a given point. The given point is called the circle's center. The constant distance is called the radius. A segment joining the center with a point on the circle is also called a radius (plural: *radii).* A circle is represented by the expression $\odot A$, where *A* is the center, or $\odot AX$, where *A* is the center and *X* is a point on the circle.

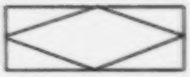
circumcenter of a triangle. The point at which the perpendicular bisectors of the sides of a triangle intersect.

circumference. The distance around a circle.

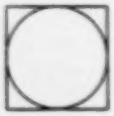
circumscribed. A term describing a figure drawn around another figure.



Circle circumscribed around a triangle



Rectangle circumscribed around a rhombus



Square circumscribed around a circle

collinear points. A set of points that lie on the same line.

concave polygon. A polygon for which at least one line containing a side of the polygon intersects another side of the polygon.

concentric circles. Circles in a plane with the same center even though the radii may differ.

concurrency. The intersection of two or more lines at the same point.

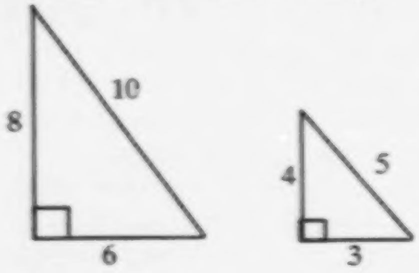
conditional. A statement that can be written in the form "if *S*, then *T*," where *S* and *T* are statements. The statement *S* is called the *hypothesis, condition,* or *antecedent*; *T* is called the *conclusion* or *consequent*.

congruent angles. Angles with the same measure.

congruent segments. Segments with the same length.

congruent polygons. Two polygons having the same size and shape such that each pair of corresponding sides have the same length and each pair of corresponding angles have the same measure.

constant of proportionality. The constant k in the proportion $\frac{x}{y} = \frac{w}{z} = k$. For similar figures, *k* is the constant ratio, known as the scale factor, of corresponding sides.



contrapositive. The conditional statement "if *S*, then *T*" written as "if not *T*, then not *S*." The contrapositive exchanges the original hypothesis and conclusion and negates them. If a statement is true, its contrapositive is true. See also *conditional, converse, biconditional* and *inverse*.

converse. The conditional statement "if *S*, then *T*" written as "if *T*, then *S*." The converse interchanges the original conditional's hypothesis and the conclusion. The truth of the converse is independent of the truth of the original conditional statement.

convex polygon. A polygon for which any line containing a side of the polygon does not intersect another side of the polygon.

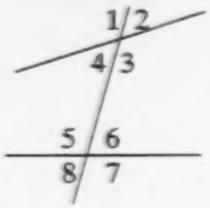
coordinates. An ordered pair of numbers naming a point on the coordinate plane.

coordinate plane. A plane in which a one-to-one correspondence exists between the set of points on the plane and the set of ordered pairs of real numbers.

coordinate system. A set of points with axes as a frame of reference. A two-dimensional system uses two perpendicular axes to define a plane. Points in a plane are represented by ordered pairs of real numbers (a, b) with a as its x-coordinate and b as its y-coordinate.

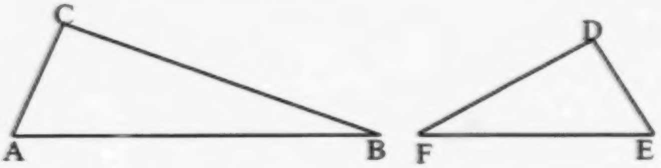
coplanar. A term describing two or more objects lying in the same plane.

corresponding angles. (1) The angles on the same side of the transversal (that is, on the left or right side) and on the same side of two lines (that is, above or below them). See also *transversal.*

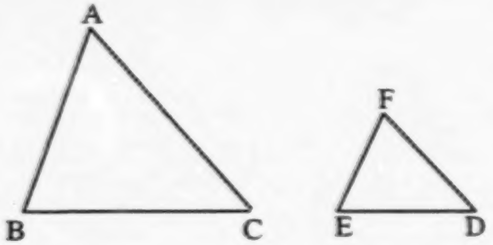


- $\angle 1$ corresponds to $\angle 5$
- $\angle 2$ corresponds to $\angle 6$
- $\angle 3$ corresponds to $\angle 7$
- $\angle 4$ corresponds to $\angle 8$

(2) A pair of angles, one from each of two similar polygons, which have the same measure. If the two similar figures are also congruent, each pair of corresponding angles lies between two pairs of congruent sides. In the figures below, $\Delta ABC \sim \Delta EFD$. $\angle ACB$ corresponds to $\angle EDF$, $\angle BAC$ corresponds to $\angle FED$, and $\angle ABC$ corresponds to $\angle EFD$.



corresponding sides. Given two similar polygons, the sides that are in the same relative position in the figures. In the figures below, $\Delta ACB \sim \Delta FDE$, $\overline{AB}$ corresponds to $\overline{FE}$, $\overline{AC}$ corresponds to $\overline{FD}$, and $\overline{CB}$ corresponds to $\overline{DE}$.



diagonal of a polygon. A segment from one vertex of a polygon to a nonadjacent vertex.

diameter. A chord that contains the center of a circle.

distance between a point and a line. For any given line and a point not on the line, the length of the perpendicular segment from the point to the line.

distance formula. The calculation of the distance between two points. Given the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the distance between the points is equal to

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

endpoint. A point which marks the end of a figure such as a segment, ray, or arc.

equiangular triangle. A triangle with all three angles of equal measure.

equidistant. Two or more points located the same distance from a given point.

equilateral triangle. A triangle with three sides of the same length.

Euler Line. In a triangle, the line on which the circumcenter, the orthocenter, and the centroid all lie.

Euler Points. In a triangle, the points designating the circumcenter, the orthocenter, and the centroid.

exterior angle of a polygon. An angle that forms a linear pair with one of the interior angles of the polygon.

hexagon. A polygon with six sides.

horizontal line. A line parallel to the plane at the horizon. In the coordinate plane, this line is usually parallel to or on the *x*-axis.

hypotenuse. In a right triangle, the side opposite the right angle.

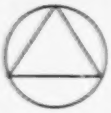
image. If *A* is mapped onto A', A' is called the image of *A*. The pre-image of A' is *A*.

image point. A point on the image of a transformation.

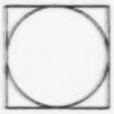
incenter of a triangle. The center of a circle inscribed in a given triangle. It is found by locating the intersections of the angle bisectors of the triangle.

indirect proof. A proposition proven to be true by supposing it to be false and then demonstrating that it cannot be false, for if it were, some accepted or previously proven facts would be contradicted.

inscribed. A term describing a figure drawn inside another figure.



Triangle inscribed in a circle



Circle inscribed in a square

Rhombus inscribed in a rectangle

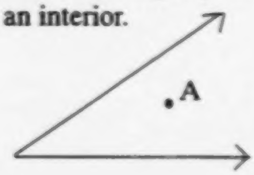
inscribed angle. An angle whose vertex lies on a circle and whose sides contain chords of the circle.

integer. One of the members of the set {. . ., -3, -2, -1, 0, 1, 2, 3, . . .}.

intercepted arc. An angle intercepts an arc if and only if the following conditions exist: (1) The endpoints of the arc lie on the angle. (2) All points of the arc, except the endpoints, are in the interior of the angle. (3) Each side of the angle contains an endpoint of the arc.

intercept. A point at which a graph crosses the *x*-axis or *y*-axis.

interior of an angle. A point is in the interior of an angle (of less than 180°) if and only if it lies on a segment connecting points on the two sides of the angle (and is not one of those endpoints). Angles with measures of 0° or 180° do not have an interior.



intersection. An element or set of elements common to two or more sets. Lines, segments, or rays intersect if they have a point in common.

inverse. The conditional statement "if *S*, then *T*" written as "if not *S*, then not *T*." Find the inverse of a conditional statement by negating the hypothesis and the conclusion. The truth of an inverse is independent of the truth of the conditional.

isometry. A transformation that preserves length and angle measure.

isosceles trapezoid. A trapezoid whose legs are congruent.

isosceles triangle. A triangle with at least two congruent sides. The congruent sides are called legs and the noncongruent side is called the base.

kite. A quadrilateral with each side congruent to exactly one adjacent side.

legs of a right triangle. The perpendicular segments of a right triangle.

legs of an isosceles triangle. The congruent sides of an isosceles triangle.

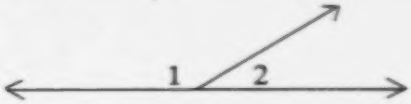
legs of a trapezoid. The nonparallel sides of a trapezoid.

length. The distance between two points expressed in terms of a unit of measure.

length of an arc. The measure of an arc which represents a fraction of the circumference of a circle.

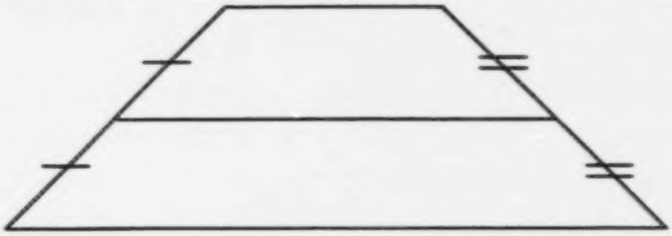
line. An assumed or undefined term in plane geometry. It can be thought of as a set of points extending infinitely in opposite directions. It is related to other undefined terms by axioms such as "Two points determine a line" and "The intersection of two planes is a line."

linear pair of angles. A pair of adjacent angles whose non-common sides are opposite rays.

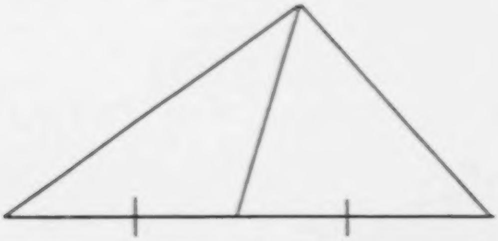


measure of an arc. The measure of the central angle that intercepts the arc.

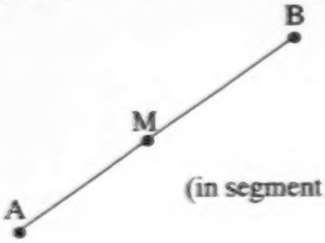
median of a trapezoid. The segment that joins the midpoints of the legs of a trapezoid.



median of a triangle. A segment joining a vertex of a triangle to the midpoint of the side opposite that vertex.



midpoint of a segment. The point of a segment equidistant from its endpoints



(in segment *AB*, with the midpoint *M*, $AM = MB$)

mirror line. A line over which a transformation reflects figures.

obtuse angle. An angle with a measure greater than 90° but less than 180°.

obtuse triangle. A triangle with one obtuse angle.

octagon. A polygon with eight sides.

opposite angles of a quadrilateral. A pair of angles in a quadrilateral, that do not have a common side.

opposite rays. Rays $\overrightarrow{EF}$ and $\overrightarrow{ED}$ are opposite rays if and only if *E* is between *F* and *D*.

opposite vertices (of a rectangle). The endpoints of the diagonals or a pair of vertices that are not adjacent.

origin. On a coordinate plane, the point corresponding to (0,0), the intersection of the two axes.

orthocenter. The point at which the altitudes of a triangle intersect. This point lies on the Euler Line of the triangle.

parallel (lines). Lines that are coplanar and do not intersect.

parallel segments. Segments that lie on parallel lines.

parallelogram. A quadrilateral in which each pair of opposite sides are parallel.

pentagon. A polygon with five sides.

perimeter of a polygon. The sum of the lengths of the sides of a polygon.

perpendicular. Two lines intersecting at right angles. Rays or segments that intersect and form right angles are said to be perpendicular. (the notation for perpendicular lines $\overleftrightarrow{AB}$ & $\overleftrightarrow{CD}$ is $AB \perp CD$)

perpendicular bisector of a segment. A line, segment, ray, or plane that is perpendicular to a given segment at that segment's midpoint. Any point on the perpendicular bisector is equidistant from the ends of the segment.

plane. An undefined term. It can be thought of as a flat surface that extends without bounds. It is linked to other terms by such axioms as "Three noncollinear points determine a plane," or "The intersection of two planes is a line."

point. An undefined term in plane geometry. A point can be thought of as an exact, dimensionless location.

point of intersection. A point common to two or more sets of points.

polygon. A closed figure made of three or more segments which intersect only at their endpoints.

proportion. An equation of the form $\frac{a}{b} = \frac{c}{d}$ expressing the equivalence of two ratios.

proposition. A concept or principle to be proven.

proof. A formal structure using axioms and theorems to show the validity of a concept or principle.

Pythagorean Theorem. The sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse if and only if the triangle is a right triangle. See also *hypotenuse*.

quad [illegible] olygon with four sides.

radiu [illegible] ent whose endpoints are the center of the circle and a point on the circle; the distance from the center to a point on the circle is the measure of the radius.

ratio. A quotient such as $\frac{a}{b}$, where b is not 0. Other notations for ratio include $a:b$, or a to b.

rational number. A number written in the form $\frac{a}{b}$ such that a and b are integers and b is not 0.

ray. A half-line defined by fixing an initial point on a line and containing the infinite number of points on one side of it.

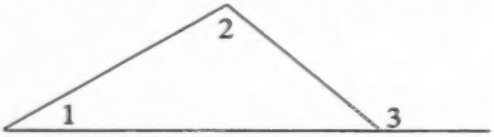
real numbers. The union of all the rational and irrational numbers; the set of all numbers corresponding to points on a number line.

rectangle. A parallelogram with one right angle.

reflection. A transformation that "flips" figures in the plane "over" a line. The line is the axis of the reflection or *mirror line.* Corresponding points on the image and pre-image are the same distance from the mirror line. The mirror line is the perpendicular bisector of each segment that connects corresponding pre-image and image points. (The mirror line is a fixed line; that is, the reflection image of any point P on the mirror line is P itself)

regular polygon. A polygon in which all the sides have the same length and all the angles have the same measure.

remote interior angles of a triangle. The angles of a triangle not adjacent to a given exterior angle. In the triangle below, angles 1 and 2 are remote interior angles to angle 3.



rhombus. A quadrilateral in which all the sides are congruent.

right angle. An angle that measures 90°.

right triangle. A triangle with one right angle. The sides forming the right angle are called *legs.* The side opposite the right angle is called the *hypotenuse.*

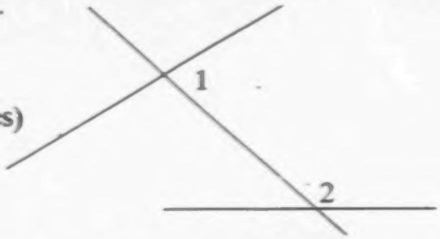
rise. A term used in describing the slope of a line that is set on coordinate axes. This term refers to the vertical distance on the y-axis, between two coordinate points.

rotation. A transformation that turns figures in the plane through a given angle (the angle of rotation) about a given point (the center of rotation). Each image point is on a circle having its center at the center of rotation and passing through the corresponding pre-image point. Further, the arcs determined by image points and their corresponding pre-image points on these concentric circles all have the same (degree) measure.

run. A term used in describing the slope of a line. This term refers to the horizontal distance, on the x-axis, between two coordinate points.

same-side interior angles. A pair of angles formed by a transversal that crosses two lines, interior to the two lines, and on the same side of the transversal.

(∠ 1 and ∠ 2 are same-side interior angles)

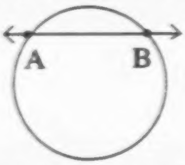


scale. A transformation in which figures in the plane are stretched uniformly outward or shrunk uniformly inward from a fixed center point. Scaling preserves shape, but may change size.

scale factor in similarity. The ratio of the lengths of corresponding sides of two similar polygons.

scalene triangle. A triangle with no congruent sides.

secant of a circle. A line, ray, or segment that contains a chord. (secant $\overrightarrow{AB}$, $\overleftrightarrow{AB}$ contains chord $\overline{AB}$)



segment. Part of a line consisting of the union of two given endpoints and all the points between them.

semicircle. The arc determined by the endpoints of a diameter of a circle.

shear. A transformation that shifts figures in the plane in opposite directions on each side of a fixed line. Corresponding image and pre-image points remain the same distance from the fixed line. A shear distorts most distances and angles yet preserves area.

similar polygons. Two polygons for which a one-to-one correspondence exists between their vertices such that each pair of corresponding angles is congruent and the ratios of corresponding sides are equal.

slope of a line. The ratio of the change in y-coordinates to the change in x-coordinates on a given line in a rectangular coordinate system. A line's slope indicates its "slant." (ie. rise ÷ run)

square. A rectangle with congruent sides. Alternately, a square is a rhombus with one right angle.

straight angle. An angle with measure 180°. Its sides are opposite rays and their union forms a line.

supplementary angles. Two angles, the sum of whose measures equals 180°.

tangent to a circle. A line that intersects a circle at exactly one point, called the *point of tangency.* Segments or rays contained in a tangent line and containing the point of tangency also are said to be tangent to a circle.

theorem. A mathematical statement that has been shown to be true. A theorem is derived from axioms and previously proven theorems.

transformation. A one-to-one function that maps points on one figure in the plane to points on another figure in the plane. The first figure is called the *pre-image,* the other, the *image.* Points on the image are called *image points;* points on the pre-image are called *pre-image points.*

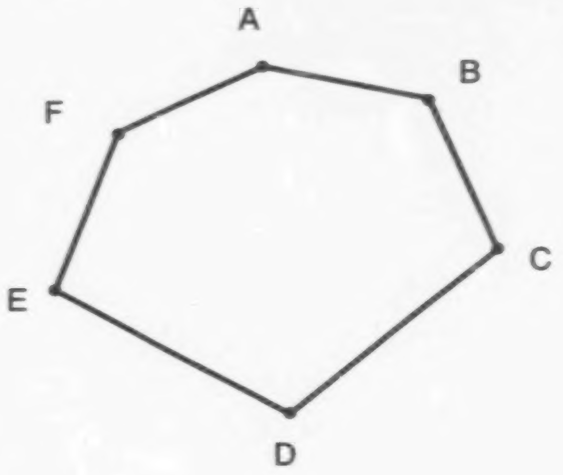
translation. A transformation that slides figures in the plane a given distance in a given direction. Segments connecting pre-image points with their corresponding image points are parallel and equal in length.

transversal. A line that intersects at least two lines at different points.

trapezoid. A quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases.* The nonparallel sides are called *legs.* Pairs of angles formed by a base and two legs are called *base angles.*

triangle. A polygon with three sides. A triangle is represented by the notation ΔABC.

vertex. The common endpoint of two rays that form an angle (plural: *vertices*).



vertex of a polygon. The common endpoints of two sides of a polygon. The points *A, B, C, D, E,* and *F* are each a vertex.

vertical angles. A pair of angles whose sides form two pairs of opposite rays.

vertical line. A line perpendicular to the plane of the horizon, usually parallel to or on the *y*-axis.

x-axis. The horizontal (usually) axis on a coordinate plane.

x-coordinate. The first coordinate of an ordered pair, also called abscissa.

y-axis. The vertical (usually) axis on a coordinate plane.

y-coordinate. The second coordinate of an ordered pair, also called ordinate.